Transshipment in supply chain networks with perishable items

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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April, 2019

Declaration of Authorship

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed. I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

Maryam Dehghani

April, 2019

"Dedicated to my inspiring husband, for his endless love, support, and encouragement."

Acknowledgements

I would like to express my sincere gratitude to my principal supervisor Professor Babak Abbasi for providing the scope of this work, his invaluable support, and motivation throughout the course of my PhD studies. This thesis would not have been possible without his insights.

I would also like to express my deepest gratitude to my co-supervisor Assistant Professor Fabricio Oliveira, for his support and sharing his knowledge and detailed review of my written work. I have been extremely fortunate to have the opportunity to learn from and collaborate with him.

It has been a great pleasure working with the faculty, staff, and students in the Business School at RMIT University during my PhD. I wish to express gratitude to all the staff in the School of Business IT and Logistics.

I am grateful to my family and friends for being a constant source of encouragement and motivation. Most of all, I would like to thank my parents and my beloved husband for their great love, encouragement and inspiration.

List of publications arising

1- Dehghani, Maryam, and Babak Abbasi. " An Age-based Lateral-transshipment Policy for Perishable Items ". International Journal of Production Economics 198, pp. 93–103.

2- Dehghani, Maryam, Babak Abbasi, and Fabricio Oliveira. " Proactive Transshipment in the Blood Supply Chain: a Stochastic Programming Approach ". Under review in Omega: The International Journal of Management Science.

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Abstract

Supply chain management is an efficient approach to managing the flow of information, goods, and services in fulfillment of customer demand. The implementation of supply chain management significantly affects the cost, benefit level, and quality. Over the past decades, multiple strategies for effective supply chain management have been developed in both academia and industry. One such strategy is named lateral transshipment which allows movement of stock between locations at the same echelon level or even across different levels.

Although transshipment has been considered in the literature for a long time, there has been limited studies of transshipment for perishable items, most likely because of the complex structure of perishable inventories. The analysis of perishable-inventory systems has been considered in numerous articles because of its potential application in sectors such as chemicals, food, photography, pharmaceuticals, and blood bank management.

Blood services in Australia rely on voluntary, non-remunerated donors to satisfy the demand for blood. Blood services confront ongoing challenges in providing an adequate supply of blood and blood products. One of the powerful tools that could improve the efficiency of the blood supply chain is lateral transshipment.

This thesis presents three models that have application in the transshipment of perishable items such as blood.

The first model (presented in Chapter 2) outlines the development of a new transshipment policy for perishable items, to enhance supply chain performance. A Poisson-distributed customer demand is assumed and the effect of reactive transshipment on expected costs are evaluated. A heuristic solution is developed, using partial differential equations to compute performance measures and cost function. The performance of this model is evaluated through a numerical study. The results indicate that this transshipment policy is effective under lost-sale and backordering scenarios. In addition, the performance of the suggested transshipment policy is compared with the

current transshipment policy that is practiced in some Australian hospitals. The results suggest that by setting the optimal threshold, a significant cost saving could be obtained with the same average issuing age of the current policy.

The second model (presented in Chapter 3) considers a finite-horizon multi-period inventory system with one main hospital connected to several smaller hospitals. The hospitals face random demand and small hospitals are allowed to transship to the big hospital to mitigate their wastage. The problem is formulated as an infinite-horizon dynamic programming model. The objective of this model is to determine an optimal ordering and transshipment policy that minimizes the total expected cost. An approximate dynamic programming (ADP) model is used to approximate the value function with a linear combination of basis functions, using column generation to cope with the course of dimensionality. The numerical results suggest that considerable cost saving can be achieved by using an ADP model.

The third model (presented in Chapter 4) proposes a proactive transshipment policy for a network of hospitals with uncertain demand. At the beginning of each review period, each hospital makes decisions on the quantity to order from a central blood bank and to transship to other hospitals. The problem is formulated as a two-stage stochastic programming model where the Quasi-Monte Carlo (QMC) sampling approach is used to generate scenarios and the optimal number of scenarios is determined by conducting stability tests. The performance of the developed model is evaluated through numerical experiences. The numerical results indicate significant potential cost savings in comparison with the current policy in use and the no-transshipment policy.

Chapter 1

Introduction

Blood is a precious commodity that has a strictly monitored and finite shelf life. Typically, 7 to 8% of human body weight is attributable to blood. Human blood is a scarce, useful resource that can only be generated by human beings; at this stage, no chemical process can be utilized to produce blood.

Blood has several vital functions in the human body, such as transporting oxygen and nutrients to the various cells and tissues of the body and disposing of ammonia, carbon dioxide, and different waste products. Furthermore, it performs a critical role in the immune system and regulation of body temperature. Blood has several different components; four of the most vital ones are white cells, red cells, plasma, and platelets.

Red blood cells are the most numerous cells in blood, generally constituting 40 to 50% of the whole blood volume. They contain a protein known as hemoglobin, which moves oxygen from the lungs to all of the body's tissues. Red blood cells are formed regularly in the bone marrow from stem cells, called hemocytoblasts, at a rate of approximately 2 to 3 million cells every second. Red blood cells are given to the patients suffering from kidney failure, sickle cell anemia, and gastrointestinal bleeding. Red blood cells have an average shelf life ranging from 21 to 42 days, depending on the country.

White blood cells, also referred to as leukocytes, are part of the immune system, and are responsible for protecting the body against disease. White blood cells are considerably less common than red blood cells and form a very small percentage of the blood's volume: commonly, almost 1%.

Plasma is a yellowish liquid component of blood, with a one-year shelf life. It mostly consists of water, with ions, proteins, and nutrients. Plasma is transfused into patients with clotting disorders, and trauma and burn injuries.

Platelets are the most perishable blood component, with a shelf life of five days. Platelets help to form clots to stop bleeding and are consequently required for patients with bleeding disorders. Platelets are a vital component of today's treatment plans, including those associated with chemotherapy, bone marrow transplants, radiation treatment, and organ transplants.

1.1 Blood supply chain

The blood supply chain includes the processes of collecting, testing, processing, and distributing blood and blood products from donors to recipients. The first echelon of the blood supply chain is collection, which involves the procurement of blood and blood products. This stage is concerned with acquiring the quantity of blood and blood products required to fulfill demand. Normally, blood is collected at fixed or mobile locations of donor centers. Decisions made at this stage are mostly associated with the management of blood collection: location and capacity decisions, methods of collecting, and donor management.

In the next stage, production, a unit of blood is moved to the blood center for testing and breaking down into components. The objective of this stage is to replenish inventories of blood products during normal and emergency periods. Decisions made at this stage usually refer to the determination of location and capacities, staff allocation, production plans, and daily planning such as scheduling of staff, timetabling, and scheduling for testing.

The third echelon of the blood supply chain is inventory. New methods to investigate inventory policies for blood products have been presented from 1960 onward (Osorio, Brailsford, and Smith (2015)). The specific characteristics of blood products, such as their perishability, significantly increase the complexity of creating an inventory and have inspired many theoretical developments that have had extensive application beyond the blood supply chain. Decisions in this stage are associated with network design, definition of inventory policies, and daily quantities to order.

The ultimate echelon of the blood supply chain is distribution, which includes the regional transfer of blood products from one blood center to another, when one location has a shortage while another location has stock on hand. In addition, the distribution stage includes the internal transfer of blood from a hospital blood bank to the patient. The objective is to deliver the correct amount of the appropriate product to the patient whenever it is required. Decisions in this stage are related to choosing of types of vehicles, capacity, packing, transshipments between different points, and satisfying time constraints.

1.2 Problem statement and motivations

The characteristics of blood units, such as perishability and limited donor population, makes the management of their supply chain a challenging task. Blood is sourced from a semi-unpredictable supply, as it is completely reliant on donations and as yet, cannot be produced artificially. In 2011, the American Blood Organization stated that 60% of the United States' population is eligible to donate blood but only about 5% actually does so (America's Blood Centers). The situation is similar in Australia. According to the Australian Red Cross Blood Service, only 3.3% of Australians donate blood, while 1 in 3 Australians will require a blood transfusion in their lifetime. The percentage of blood donors in the population is even smaller in developing countries (Zhou, Leung, and Pierskalla (2011)). Nagurney and Dutta (2018) noted that the average donation rate is significantly lower in developing countries than in developed countries.

As well as the uncertain nature of supply and the perishable characteristics of blood, the demand for blood products is variable, typically with higher demand during weekdays. While there may be detailed information available about patients who require blood and planned surgeries, some demand, such as that from accidents and trauma incidents, emerges randomly. Moreover, the demand for blood is growing. The aging population in many countries, such as Australia, Canada, and the USA, has caused an increased demand for advanced medical treatments that require blood transfusions. Further, the demand for blood occurs at hospitals, which increases the complexity of having a large network of demand points in the supply chain. These facts mean that balancing the supply and demand of blood in a highly efficient manner requires careful planning that considers several angles of uncertainty.

This combination of the perishability of blood and the uncertain nature of the demand imposes strict limitations , which substantially enhance the risk of shortages and wastage. Keeping the balance between shortage and outdated blood products is a major challenge in the management of the blood inventory at a hospital. When hospitals are confronted with a blood shortage, many medical operations must be postponed or cancelled. Recently, blood shortage has occurred frequently, which seriously affects the health of people and can put human lives at great risk (Ghandforoush and Sen (2010)). Conversely, storing an excessive number of blood units increases wastage, which is not only an economic problem but also has a normative social effect, as wasting a unit of blood is a waste of the donors' effort, time, and contribution. Wastage can occur all along the blood supply chain; however, Stanger et al. (2012a) discovered that wastage in hospitals is considerably higher than wastage in blood centers, which is undesirable, as an outdated item at one hospital could have been utilized at another hospital to save a person's life. Blood inventory management is, therefore, a trade-off between shortage and wastage.

Lateral transshipment is a powerful tool for ensuring that perishable items are available in an efficient way at the demand points. Lateral transshipment consists of stock movements between locations in the same echelon of an inventory system. This approach conventionally balances stock by reallocating the network's inventory. When operating within a network of facilities (such as hospitals), transshipment could be an effective way to adjust the discrepancy between the current/future demand and the inventory of blood units among hospitals. For instance, Stanger et al. (2013) conducted a survey on the effect of transshipment in the United Kingdom's blood supply chain and showed that transshipment of blood between hospitals enhances flexibility in blood supply management and diminishes the number of outdated units. Moreover, transshipment supports hospitals in dealing with shortages more efficiently by using nearby hospitals' stocks. Further, Abbasi, Vakili, and Chesneau (2017) indicated that blood transshipment in a large network of hospitals could improve the performance measures of blood supply chains and possibly help to reduce the shelf life of red blood cells, to ensure that patients receive fresher units.

Although transshipment has been considered in the literature, few studies have considered the effect of relying on lateral transshipment when managing inventories of perishable items, and of blood in particular. Therefore, this research investigates the effect of lateral transshipment on the performance of the blood supply chain.

1.3 Research aim and objectives

The aim of this research is to develop a transshipment policy for perishable inventories, in particular blood inventories, and to investigate the effect of transshipment on shortage and the costs of having outdated supplies. This research addresses the following questions:

- How can transshipment improve the performance of the supply chain for perishable items?
- How can the optimal transshipment policy for perishable items be obtained?
- What are the criteria for designing a transshipment policy for perishable items?
- Which transshipment structure (one-way or two-way) is more appropriate for perishable items?

These questions gave rise to the following objectives for this research:

- to establish a mathematical model to control the inventory for perishable items
- to develop a mathematical model that would give the best transshipment policy
- to compare two transshipment policies (one-way and two-way) and find which one is more appropriate for perishable items.

1.4 Thesis structure

Three models were developed in this research. The first model (presented in Chapter 2) introduces a new policy for transshipment, based on transshipping units from small hospitals to large hospitals. Transshipment in blood supply chain is often based on the age profile of blood units in hospitals. However, decisions such as the age threshold are made empirically and are fixed for all hospitals. This new transshipment policy for perishable items was based on the age of the oldest item in the system, to improve supply chain performance. The suggested policy defined a age threshold (k) and outlined a transshipment policy based on this age threshold. The age threshold is a decision variable that can be determined for each location (e.g., hospital) and is designed to reduce outdate and shortage. The model assumes a Poisson-distributed customer demand and a $(S-1, S)$ inventory system, which is a replenishment order that occurs immediately after a demand has taken place. The proposed model has applications for transshipping blood units between hospitals. A heuristic solution using partial differential equations to compute performance measures and cost function is developed. The results demonstrated that this transshipment policy was effective under various circumstances, such as lost-sale and backordering. The performance of the suggested transshipment policy is compared with the transshipment policy that is currently practiced in some Australian hospitals (which is an aged-based proactive transshipment policy with an empirically set threshold). Unfortunately, in several cases, the demand for products is not directly observable. Therefore, two other models are developed to analyze the effect of transshipment on performance measure with random demand.

The second model proposes a proactive transshipment policy to control the inventory in the blood supply chain. A finite-horizon multi-period inventory system is considered, with one main hospital connected to several smaller hospitals. The objective is to minimize the total expected cost over the time horizon by determining the optimal orders and proactive transshipment for each period. The cost function comprised of ordering, transshipment, inventory, shortage, and outdate costs. The non-negative stochastic demand is considered, with general distribution over the time horizon for each hospital. In this model, the hospitals maintain their own blood inventory, supplied from a central blood bank (CBB). In addition to receiving supplies from a CBB, some small hospitals (in a predefined network) could transship blood units to the main hospital, to reduce outdates. The problem is formulated as an infinite-horizon dynamic programming model.

Dynamic programming offers an integrated approach to solving problems of stochastic control. A key aspect in the methodology is the cost-to-go function, which can be acquired via solving Bellman's equation. The domain of the cost-to-go function is the state space of the system to be controlled, and dynamic programming algorithms calculate and store a table including one cost-to-go value per state. Unfortunately, the size of a state space generally grows exponentially in the number of state variables, which is known as "the curse of dimensionality." This phenomenon causes the dynamic programming to be intractable when confronted with the problems of practical scale. Because of the curse of dimensionality in the state and action space of the problem, an approximate dynamic program is used to approximate the value function with a linear combination of basis functions, using column generation. The performance of the proposed model is evaluated through a numerical study. This model is presented in Chapter 3.

The third model (presented in Chapter 4) proposes a proactive transshipment policy to avoid future shortages and to mitigate wastage in the blood supply chain. A network of hospitals with uncertain demand is considered in which each hospital makes decisions on the quantity of orders from a CBB and transshipment in each review period. The problem is formulated as a two-stage stochastic programming model, using the Quasi-Monte Carlo (QMC) sampling approach to generate scenarios, with the optimal number of scenarios determined by conducting stability tests. Extensive numerical experiments are conducted to evaluate the performance of the proposed model and investigated the potential benefits of the outlined proactive transshipment. The developed model is compared with the current policy practiced in some hospitals in Australia and the no-transshipment policy. The results demonstrate the superiority of the proposed model over the current policy and the no-transshipment policy, with significant costs savings and reduced outdate rates.

Chapter 5 presents the conclusions of this research and suggests recommended directions for future research.

Chapter 2

An Age-based Lateral-transshipment Policy for Perishable Items

2.1 Introduction

The management of the inventory of perishable products is of great concern in many sectors. One of these sectors is blood banks. In general, inventory control of blood products is very challenging because all blood products have restricted shelf lives. Stock control becomes a trade-off between shortage and wastage. As the supply of blood is quite irregular and expensive, outdates are undesirable. However, a shortage of blood products can increase mortality rates in the community. The aim of blood banks and hospitals is to minimize shortages and outdates of all blood products.

This current study was motivated by the simple policy practiced in some hospitals in Australia, whereby a small hospital can agree to transship units that are older than an agreed threshold only to a large hospital (Abbasi, Vakili, and Chesneau (2017)). The motivation behind this policy is that because of the higher demand at the large hospital, the transshipped blood is likely be used at the large hospital, rather than expiring at the small hospital. Note that the above policy is a proactive policy. Although it seems logical, the identification of the threshold is empirically decided .

This chapter introduces a new policy for transshipment that is based on transshipping units from small hospitals to large hospitals. The suggested policy defines a age threshold (k) and outlines a transshipment policy based on this age threshold. The age threshold is a decision variable that can be determined for each location (e.g., hospital) and is designed to reduce outdates and shortages. The policy is based on the idea that when demand occurs at a particular location and that location is out of stock, transshipment can be requested if the age of the oldest item in the system is greater than k; otherwise, an emergency shipment should be requested from the supplier (i.e., the blood bank). Similar to the policy used in some hospitals in Australia, this new transshipment policy is an age-based policy; however, is reactive, as a transshipment decision is triggered when one of the hospitals faces a shortage. In addition, the age threshold in the new policy is optimally decided.

The new model assumes a Poisson-distributed customer demand and an $(S-1, S)$ inventory system. Given that blood is not an ordinary product, daily blood demand is uncertain. The literature supports the assumption of a Poission-distributed demand. For instance, Based on data from a hospital, Elston and Pickrel (1963) found that demand at a blood bank is a Poisson random variable. Perry and Posner (1990), Kopach, Balcıoğlu, and Carter (2008), Haijema, Wal, and Dijk (2007), Haijema et al. (2009), and Duan and Liao (2013) demonstrated that daily demand will be a Poisson distribution if the size of a requisition is one unit. Blake et al. (2013a) assumed a Poisson distribution for demand at hospitals for each day of the week to evaluate network inventory policies in Canada.

$(S-1, S)$ policy is regularly utilized as a stock-control policy for expensive, low-demand products that have moderately short lifetimes. This policy is known to be optimal under general conditions within an infinite-lifetime circumstance (see Schultz (1990)). Some hospitals in the United Kingdom follow a base-stock policy when designing their blood-ordering policy (Stanger et al. (2012b)). Hence, in this new model, an $(S-1, S)$ policy is appropriate. The objective function of this model incorporates the holding, outdate, shortage, and transshipment costs. The model is solved to determine the base-stock levels and the parameters of the transshipment policy, such that the expected cost would be minimized. When compared with never using transshipment or always requesting emergency orders when stock-outs occur, this transshipment policy is found to provide a cost-performance guarantee.

The remainder of this chapter is organized as follows. In the next section, a review of the relevant literature is presented. In Section 2.3, a description of the new model is provided with a list of the notations. Section 2.4 introduces the new transshipment policy, formulating and solving the new model by using a partial differential equation for a lost-sale scenario. In Section 2.5, the model is extended to consider backlog.

Section 2.6 presents the numerical results to evaluate the model. Finally, Section 2.7 presents a concluding discussion.

2.2 Literature review

This study is related to the literature of two research streams: reactive transshipment and perishable items.

2.2.1 Reactive transshipment

Transshipment, which is known as a means of improving the performance of an inventory system and supply chain, has been the subject of some research attention. The first study on transshipment was conducted by Krishnan and Rao (1965), who assumed a periodic-review policy in a single-echelon, single-periodic setting, which has negligible transshipment times. They minimized the cost through transshipment once all demand was known.

Paterson, Teunter, and Glazebrook (2012) provided an overview of lateral-transshipment policies and models. They classified previous research into reactive and proactive transshipment, which differ in the timing of transshipment: reactive transshipment occurs after observing demand and can occur at any time; proactive transshipment occurs at fixed points in time before demand realization.

The majority of the models of reactive transshipment (Herer, Tzur, and Yücesan (2006), Axsäter (1990), Banerjee, Burton, and Banerjee (2003), Burton and Banerjee (2005), Zhang (2005), Wee and Dada (2005), Yang and Qin (2007), Zhao and Atkins (2009), and Tang and Yan (2010)) have assumed that transshipment time is negligible. This assumption affects the nature of the transshipment policy and reduces the complexity of the model.

In this study, the focus is on designing transshipment policies for perishable items. While few studies have analyzed transshipment policies in perishable-inventory systems, related research is reviewed here to provide insight into some of the assumptions and attributes of this study's inventory system.

Moinzadeh and Schmidt (1991) considered an $(S-1, S)$ ordering policy with a Poisson demand and two suppliers, and proposed two modes of replenishment: normal and emergency. The decision for placing emergency orders was based on time passed for outstanding orders. The same approach is used, but considering perishable items, when designing the transshipment policy for this research.

Axsäter (2003) presented a model with a single-echelon inventory system with multiple warehouses, each facing a Poisson demand and using continuous review (R, Q) policies. The author proposed decision rules for transshipment based on information about the system state (e.g., remaining delivery times for outstanding orders) to minimize the expected cost. When demand occurs at a warehouse, the decision rules determine whether the demand should be covered by a lateral transshipment from another warehouse and if so, from which warehouse. Yang et al. (2013) considered a single-item inventory model with two echelons, in which the lateral transshipment time is not negligible, and introduced a measure of customer-oriented service, which means that a demand is considered met if it is responded to within a certain time window. They assumed that demand processes are independent Poisson processes. They approximated the performance measures using queuing models and optimized base stock by minimizing the expected inventory cost. Tai and Ching (2014) presented a two- echelon inventory system with one central warehouse and some local warehouses. By allowing transshipment among the local warehouses, they developed a Markovian queuing model that minimized the total cost. Liao et al. (2014) developed a model to compare the optimal inventory decisions under emergency-order and lateral-transshipment strategies when a stock-out occurs.

More recently, Howard et al. (2015) introduced a new policy, known as (S, T), to control a two-echelon inventory system with a central warehouse, a support warehouse, and some local warehouses. The policy determines stock levels and strategies for requesting emergency shipments. When a local warehouse is out of stock and demand occurs, the demand is backordered if there is a regular replenishment order arriving within a set threshold time. If the order does not arrive within T time units, an emergency shipment is requested. The request first goes to the support warehouse; if there is no stock in the support warehouse or stock does not arrive within its own threshold time, an emergency shipment is requested from the central warehouse instead. Howard et al. (2015) used queuing theory and provided a decomposition technique for optimizing the policy parameters to achieve cost-efficient policies.

Paterson, Teunter, and Glazebrook (2012) proposed an enhanced reactive approach that incorporates a proactive element. In their model, all locations apply a continuous review (R, Q) ordering policy. Whenever a location encounters a shortage, an algorithm determines the most cost-efficient quantity of transshipment and the location from which to transship. The policy allows more stock to be transshipped than is required to satisfy the immediate shortage and to adjust future risk. They demonstrated that this policy reduces operating costs, when compared to a purely reactive approach, as well as reducing the requirement for safety stock and improving service levels.

Olsson (2015)'s research was closely related to the work of this current research in terms of the modeling procedure. The author considered a single-echelon inventory system with two locations with a Poisson demand, applying the same approach as Schmidt and Nahmias (1985). Olsson (2015) suggested a transshipment policy based on the timing of all outstanding orders and used the concepts of doubly stochastic Poisson processes to approximate the demand at each location, and partial differential equations to solve the model. However, there is a significant difference between Olsson (2015)'s transshipment policy and the one developed in this research. First, his work was not in the domain of perishable inventory. Secondly, from the perspective of modeling assumptions, Olsson (2015) assumed that a lateral transshipment is triggered only if the transshipment lead time is shorter than the residual lead time for the oldest item. The model developed in this research assumed zero lead time for transshipment and considered the age of items to accommodate the perishability characteristic, using the age threshold as a decision variable.

Patriarca, Costantino, and Di Gravio (2016) considered a multi-echelon, multi-item system and used a genetic algorithm to determine optimal stock levels when unidirectional lateral transshipment is allowed. Zhao et al. (2016) designed a new e-commercial model (online-to-offline) and analyzed the effect of lateral transshipment on a dual-channel supply chain.

A stream of related literature has been dedicated to designing the links and structures for transshipment between different locations. For example, Lien et al. (2011) introduced chain configurations in transshipment literature, with each location connected to two locations, forming a connected loop. They considered an order-up-to policy for replenishment orders and random demand for retailers. They analytically proved that chain configuration is more cost efficient than group configurations.

Cheong (2013) considered a single perishable product model with a two-period lifetime (old or fresh). This study assumes a stochastic demand and allows for transshipment among multiple retailers. Cheong (2013) considered a single-period horizon problem that determined order quantities of fresh items and transshipment quantities of old items to minimize the total expected cost.

Wang and Ma (2015) explored the quantitative formulations for properties of inventory structure considering the age of blood. They assumed that each blood bank applies an (s, S) ordering policy, in which an order is placed to bring the inventory position up to the level S when the inventory position drops below the level s. They proposed two selection methods (first-in-first and last-in-first) to determine which items should be transshipped from the rescue blood bank when the affected bank faced shortages for a certain period (e.g., because of a natural disaster) and demonstrated that first-in-first transshipment could better reduce the outdate rate. They assumed that the excessive inventory at the rescue bank could meet the demand of the affected bank.

2.2.2 Perishable inventory

This section briefly reviews the research on perishable inventory that relates to this current study. Stock control of perishable items is extremely difficult. Significant difficulties arise from indeterminate demand of the products and their restricted shelf life. Nahmias (1982) provided the first review of the literature on perishable products; later, Goyal and Giri (2001) and Bakker, Riezebos, and Teunter (2012) presented a comprehensive review of the literature on this topic. More recently, Osorio, Brailsford, and Smith (2015) and Dillon, Oliveira, and Abbasi (2017) conducted studies in the domain of blood inventory management.

The modeling approach used in this study was close to the work conducted by Schmidt and Nahmias (1985), who developed the first model with non-zero lead time in the context of perishables. Schmidt and Nahmias (1985) considered $(S-1, S)$ policies for a perishable-inventory system. They modeled demand by a stationary Poisson process and determined the stationary distribution of the S-dimensional stochastic process as the time elapsed since the last S orders were placed, using this distribution to drive the expected cost as a function of S. Moinzadeh (1989) used the same approach but in a different context (non-perishable-inventory systems). Olsson and Tydesjö (2010) extended Schmidt and Nahmias's (1985) model by permitting backorders.

Liu and Shi (1999) analyzed an (s, S) continuous-review model for inventory systems in which items had an exponential random lifetime. Liu (1990) studied an (s, S)

continuous review inventory system with a Poisson demand and exponential lifetime distribution. Liu (1990) derived the stationary probability distribution of the inventory level and other system performance measures to determine the cost function. In the model proposed by Liu (1990), lead time was considered as zero, and backlogs were permitted.

Tekin, Gürler, and Berk (2001) introduced a new replenishment policy named (Q, r, T). This policy orders when inventory drops to r or when items exceed T units of age. Based on the new age-based policy, they derived a lost-sale-inventory model for perishable items.

Haijema (2013) presented a new ordering policy (a (s, S, q, Q) policy) for blood platelets, which was a periodic review (s, S) policy with the quantity of order bounded by a minimum (q) and a maximum (Q), and formulated this policy as a periodic Markov decision problem.

Berk and Gürler (2008) considered a (Q, r) policy, where $r < Q$ with Poisson demands and lost sales. They modeled the system by using the residual shelf life as an embedded Markov process and derived the expected cost function by regarding the stationary distribution of the residual shelf life.

One way to enhance the sustainability of the blood supply chain is to centralize the location of the blood inventory and satisfy the demand from this single location. Hosseinifard and Abbasi (2017) investigated the effect of centralization at a two-echelon blood supply chain. They demonstrated that by reducing the number of hospitals that hold inventory, the outdate and the shortage is reduced. The model developed in the research was more general than that of Hosseinifard and Abbasi (2017) because it did not force any hospitals to hold zero inventory.

Nakandala, Lau, and Shum (2017) developed a lateral transshipment model for fresh food supply chains. They considered spoilage costs, purchase costs, backordering, transshipment transshipment costs, and holding costs as the cost elements and determined the quantity of transshipment by the trade-off among those cost components.

This research found no studies that have addressed the performance of continuous review inventory systems by considering the transshipment of perishable items. Thus, the principal difference between this study and the existing literature is the development of a transshipment policy in a continuous review inventory system and the analysis of the effect of lateral transshipment on supply chain performance with perishable items.

2.3 Model description

The model develop for this research involves a single-echelon inventory system with two inventory locations ($i = 1, 2$). Each of these locations applies a continuous-review-based stock policy (i.e., $(S - 1, S)$ policy), and the customer demand at each location follows an independent Poisson process with a demand rate of λ_i. Every item in the system is presumed to have a shelf life of m units of time.

In case that location i has stock on hand, demand is fulfilled using a first-in-first-out (FIFO) policy; however, if location i is out of stock and the other location j ($j \neq i$) has available stock and the age of the oldest item is above a threshold (k_j), items are transshipped from location j to meet the demand at location i. If location j is out of stock or the oldest item in location j is younger than (k_j), two situations are considered: when demand is lost (or the demand is satisfied by emergency order), and when demand can be backordered. The emergency-order cost is assumed higher than the transshipment cost; zero lead time is assumed for emergency orders in the lost-sale scenario and zero lead time is assumed for transshipment in both lost-sale and backorder scenarios. The decision variable (k_i) is referred to the threshold age at location i. If a customer's demand is satisfied from stock on hand or backordered at location i, a new item is ordered from the supplier (i.e., the blood bank) with the lead time L_i. The rationale of this transshipment policy is based on the current practice in the transshipment of red blood cells between some hospitals in Australia; some small hospitals in Australia transship units of red blood cells to large hospitals in their network once items pass a certain age. This policy reduces outdate, which is a critical factor in blood supply chains. Therefore, this new transshipment policy is based on the age of the oldest item in each location. This means that it is important to keep track of the age of all items in each location. In this model, $\zeta_1^i(t), \zeta_2^i(t), \cdots, \zeta_{S_i}^i(t)$ (S_i represents the order-up-to level at location i) is defined as the age of the items that either are already in location i or are on the way to arrive at location i at time t. Note that the aging begins once an order is shipped. $\zeta_{S_i}^i(t)$ is considered the age of the youngest item in location i and $\zeta_1^i(t)$ is the age of the oldest item in location i. Therefore, the order is $m \geq \zeta_1^i(t) \geq \zeta_2^i(t) \geq \cdots \geq \zeta_{S_i}^i(t) \geq 0$. $\zeta^i(t) = [\zeta_1^i(t), \zeta_2^i(t), \cdots, \zeta_{S_i}^i(t)]$ is defined as an S_i-dimensional stochastic process and derives the steady distribution of $\zeta^i(t)$ to develop the explicit expression of the expected cost as a function of S_i.

Several cost components are involved in this model. It allows an emergency-order cost of q_i per unit at location i and a customer waiting cost b_i per unit per period at location i if the backorder is allowed. Furthermore, there is another cost, ρ_i at location i, associated with per-unit transshipment from location j. The outdate cost per unit in location i is denoted by θ_i. It considers the holding cost h_i per unit and unit time at location i. The notations employed in this chapter are summarized as bellow.

2.3.1 Indices

S_i - Order-up-to level at location i.
L_i - Replenishment Leadtime at location i
λ_i - Demand rate at location i.
$\bar{\lambda}_i$ - Adjusted demand rate at location i.
m - Shelf life.
α_i - Demand rate at location i if the age of oldest item at location i is less than k_i.
β_i - Demand rate at location i if the age of oldest item at location i is greater than or equal to k_i.
q_i - Emergency-order cost per unit at location i.
b_i - Backorder cost per unit per time period at location i.
ρ_i - Transshipment cost per unit from location i.
θ_i - Outdate cost per unit in location i.
h_i - Holding cost per unite per time period at location i.
k_i - Threshold age at location i.
$\zeta^i(t)$ - Vector of age profile at location i at time t. Its size is S_i.
ζ^i_1 - Age of oldest item at location i.
$p_i(t, x_1, ..., x_{S_i})$ - Joint probability density of $\zeta^i(t)$ at time t at location i.
C_i - Constant value term in $p_i(t, x_1, ..., x_{S_i})$.
Δ - Very small time interval ($\Delta \to 0$).
h - Very small value ($h \to 0$).
$P_{j,i}$ - Probability of having j items in stock at location i, $j = 0, 1, 2, ...$
$P_{-d,i}$ - Probability of having d units back-ordered at location i, $d = 1, 2, ...$
$P_j(x_1 \geq k_j)$ - Probability that the age of oldest item at location j is greater than or equal to k_j.
IL_i - Inventory at location i. It can take negative values in backordering case.
$E(IL_i)$ - Expected number of items on hand at location i.
$E(A_i)$ - Average age of issued items from location i.
$E(A)$ - Average age of issued items in the system.

$P(LT_i)$ - Probability of lateral transshipment in at location i.
$P(EO_i)$ - Probability of emergency order at location i.
O_i - Outdate rate at location i.
EC - Total expected cost.

2.4 Inventory model for the designed lateral transshipment when emergency orders are allowed (lost-sale scenario)

This section describes the performance of the system under the designed transshipment policy in which emergency orders are allowed if transshipment does not occur. This is equivalent to assuming lost sale if demand is not satisfied either on-hand inventory or transshipment. The two locations are considered independent of each other, to develop explicit formulations for the expected costs. To compute the performance of this new transshipment policy, the steady-state distribution of $\zeta^i(t)$ needs to be found, as well as the marginal distribution of the age of the oldest item in each location. Then the problem can be formulated to determine the optimal values of $k_i's$ and $S_i's$.

2.4.1 Joint probability distribution of $\zeta^i(t)$

By introducing lateral transshipment into the system, the demand rate at each location is affected. When location j is out of stock and cannot satisfy demand, if location i has stock on hand and the age of the oldest item in the system is greater than k_i, transshipment from location i is realized. Therefore, let p_{0j} be the probability of the stock-out at location j; then the demand rate at location i should be adjusted to incorporate the transshipped items from location i. The adjusted demand rate at location i, becomes:

$$\bar{\lambda}_i = \begin{cases} \alpha_i = \lambda_i, & \text{if } \zeta_1^i < k_i\,, \\ \beta_i = \lambda_i + \lambda_j \cdot p_{0,j}, & \text{if } \zeta_1^i \geq k_i \end{cases} \tag{2.1}$$

Therefore, α_i indicates the demand rate at location i if the age of oldest item at location i is less than k_i and β_i indicates the demand rate at location i if the age of oldest item at location i is greater than or equal to k_i. The probability density of $\zeta^i(t)$ at time t is defined as $p_i(t, x_1, ..., x_{S_i})$. To find the stationary distribution of $p_i(x_1, \cdots, x_{S_i})$, following the procedure used in Schmidt and Nahmias (1985) and Olsson (2015), this model considers the state of the process at time $[t, t+h)$ (h is a small positive number) and derives the partial differential equation, which is satisfied by p.

The three cases presented below can be considered.

First, when $x_1 < L_i$:
In this case, the state of the process is stable because all the items are on order. Hence:

$$p_i(t+h, x_1, \cdots, x_{S_i}) = p_i(t, x_1 - h, \cdots, x_{S_i} - h). \tag{2.2}$$

By adding and subtracting terms, (2.2) can be formulated as follows:

$$\frac{p_i(t+h, x_1, \cdots, x_{S_i}) - p_i(t, x_1, \cdots, x_{S_i})}{h} + \frac{\sum_{n=1}^{S_i} p_i(t, x_1 - h, \cdots, x_{n-1} - h, x_n, \cdots, x_{S_i}) - p_i(t, x_1 - h, \cdots, x_n - h, x_{n+1}, \cdots, x_{S_i})}{h} = 0$$

Letting $h \to 0$ gives

$$\frac{\partial p_i(t, x_1, \cdots, x_{S_i})}{\partial t} + \sum_{n=1}^{S_i} \frac{\partial p_i(t, x_1, \cdots, x_{S_i})}{\partial x_n} = 0$$

In considering the stationary distribution of $p_i(x_1, \cdots, x_{S_i})$, letting $t \to \infty$ yields

$$\sum_{n=1}^{S_i} \frac{\partial p_i(x_1, \cdots, x_{S_i})}{\partial x_n} = 0. \tag{2.3}$$

Second, when $L_i \leq x_1 < k_i$. For this case, it is assumed that there is no demand on $[t, t+h)$ with probability $(1 - \alpha_i h) + o(h)$. In contrast, if h is sufficiently small, perishing will not occur on $[t, t+h)$. Therefore:

$$p_i(t+h, x_1, \cdots, x_{S_i}) = p_i(t, x_1 - h, \cdots, x_{S_i} - h)(1 - \alpha_i h) + o(h).$$

By using the same approach as in the first case, it follows that:

$$\sum_{n=1}^{S_i} \frac{\partial p_i(x_1, \cdots, x_{S_i})}{\partial x_n} = -\alpha_i p_i(x_1, \cdots, x_{S_i}) \tag{2.4}$$

Third, for the case $x_1 \geq k_i$ and using similar arguments as the second case, the equation obtained is:

$$\sum_{n=1}^{S_i} \frac{\partial p_i(x_1, \cdots, x_{S_i})}{\partial x_n} = -\beta_i p_i(x_1, \cdots, x_{S_i}) \tag{2.5}$$

α_i and β_i are defined in (2.1).

Equations (2.3) to (2.5) are simple partial differential equations.

Note that the demand process is a non-homogeneous Poisson process and the demand in the interval $(0, t_1]$ has a Poisson distribution with rate $\int_0^{t_1} \bar{\lambda}_i(x_1)dx_1$.

Proposition 1 (adapted from Proposition 1 in Olsson (2015)) The general solution to equations (2.3) to (2.5) is as follows:

$$p_i(x_1, \cdots, x_{S_i}) = \phi(x_1 - x_2, x_2 - x_3, \cdots, x_{S_i - 1} - x_{S_i}) \times e^{-\int_0^{x_1} \bar{\lambda}_i(x)dx} \tag{2.6}$$

where ϕ is a differentiable function of $S_i - 1$ variables.

Proof. Using (2.1), Equations (2.3) to (2.5) can be written as follows:

$$\sum_{n=1}^{S_i} \frac{\partial p_i(x_1, \cdots, x_{S_i})}{\partial x_n} = -\bar{\lambda}_i p_i(x_1, \cdots, x_{S_i}) \tag{2.7}$$

The following transformations can then be applied:

$t_1 = x_{S_i - 1} - x_{S_i}$,

$t_2 = x_{S_i - 2} - x_{S_i - 1}$,

$t_3 = x_{S_i - 3} - x_{S_i - 2}$,

...,

$t_{S_i - 1} = x_1 - x_2$,

$t_{S_i} = x_1$

The chain rule gives the following:

$\frac{\partial}{\partial x_{S_i}} = \frac{\partial t_1}{\partial x_{S_i}} \times \frac{\partial}{\partial t_1} = -\frac{\partial}{\partial t_1}$

$\frac{\partial}{\partial x_{S_i-1}} = \frac{\partial t_1}{\partial x_{S_i-1}} \times \frac{\partial}{\partial t_1} + \frac{\partial t_2}{\partial x_{S_i-1}} \times \frac{\partial}{\partial t_2} = \frac{\partial}{\partial t_1} - \frac{\partial}{\partial t_2}$

$\frac{\partial}{\partial x_{S_i-2}} = \frac{\partial}{\partial t_2} - \frac{\partial}{\partial t_3}$

...

$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial t_{S_i-1}} - \frac{\partial}{\partial t_{S_i}}$

Therefore, Equation (2.7) is equivalent to

$\sum_{n=1}^{S_i} \frac{\partial p_i(x_1,\cdots,x_{S_i})}{\partial x_n} = \frac{\partial p}{\partial t_{S_i}} = \frac{\partial p}{\partial x_1} = -\bar{\lambda}_i p_i(x_1, \cdots, x_{S_i})$

This yields

$\frac{\partial}{\partial x_1}(p_i(x_1, \cdots, x_{S_i}) \times e^{-\int_0^{x_1} \bar{\lambda}_i dx}) = 0$

This means

$p_i(x_1, \cdots, x_{S_i}) \times e^{-\int_0^{x_1} \bar{\lambda}_i dx} = \phi(t_1, t_2, \cdots, t_{S_i-1}) = \phi(x_{S_i-1} - x_{S_i}, \cdots, x_1 - x_2)$
then
$p_i(x_1, \cdots, x_{S_i}) = \phi(x_1 - x_2, x_2 - x_3, \cdots, x_{S_i-1} - x_{S_i}) \times e^{-\int_0^{x_1} \bar{\lambda}_i dx}$. □

$\phi(x_{S_i-1} - x_{S_i}, \cdots, x_1 - x_2)$ is a differentiable function and Schmidt and Nahmias (1985) demonstrated that $\phi(x_{S_i-1} - x_{S_i}, \cdots, x_1 - x_2)$ is constant by using the boundary conditions that describe the process when an item perishes or demand occurs. Hence, considering $\phi(x_{S_i-1} - x_{S_i}, \cdots, x_1 - x_2) = C_i$, $p_i(x_1, \cdots, x_{S_i})$ can be expressed as follows:

$$p_i(x_1, \cdots, x_{S_i}) = C_i \cdot e^{-\int_0^{x_1} \bar{\lambda}_i dx} \tag{2.8}$$

By conditioning on the age of the oldest item, Equation (2.8) can be extended to the following:

$$p_i(x_1,\cdots,x_{S_i}) = \begin{cases} C_i e^{-\alpha_i L_i}, & \text{if } x_1 < L_i\,, \\ C_i e^{-\alpha_i x_1}, & \text{if } L_i \leq x_1 < k_i\,, \\ C_i e^{-(\alpha_i k_i + \beta_i (x_1 - k_i))}, & \text{if } x_1 \geq k_i\,. \end{cases} \tag{2.9}$$

The constant C_i is obtained by integrating (2.9) over its domain and setting it to one, to satisfy the condition of being a probability density function. Thus,

$$\int_0^m \int_0^{x_1} \cdots \int_0^{x_{S_i-1}} C_i.e^{-\int_0^{x_1} \bar{\lambda}_i dx}\, dx_{S_i} \cdots dx_2\, dx_1 =$$

$$\int_0^m C_i.e^{-\int_0^{x_1} \bar{\lambda}_i dx} \cdot \frac{x_1^{S_i-1}}{(S_i-1)!}\, dx_1 = 1$$

This implies

$$\frac{1}{C_i} = \frac{e^{-\alpha_i L_i} L_i^{S_i}}{S_i!} + \int_{L_i}^{k_i} e^{-\alpha_i x_1} . \frac{x_1^{S_i-1}}{(S_i-1)!}\, dx_1 + \int_{k_i}^{m} e^{-(\alpha_i k_i + \beta_i (x_1 - k_i))} . \frac{x_1^{S_i-1}}{(S_i-1)!}\, dx_1 \tag{2.10}$$

The marginal density of the age of the oldest item can now be calculated as follows:

$$p_i(x_1) = \int_0^{x_1} \int_0^{x_2} \cdots \int_0^{x_{S_i-1}} C_i.e^{-\int_0^{x_1} \bar{\lambda}_i dx}\, dx_{S_i} \cdots dx_2 \tag{2.11}$$

Equation (2.11) can be stated more explicitly as follows:

$$p_i(x_1) = \begin{cases} C_i e^{-\alpha_i L_i} . \frac{x_1^{S_i-1}}{(S_i-1)!}, & \text{if } x_1 < L_i\,, \\ C_i e^{-\alpha_i x_1} . \frac{x_1^{S_i-1}}{(S_i-1)!}, & \text{if } L_i \leq x_1 < k_i\,, \\ C_i e^{-(\alpha_i k_i + \beta_i (x_1 - k_i))} . \frac{x_1^{S_i-1}}{(S_i-1)!}, & \text{if } x_1 \geq k_i\,. \end{cases} \tag{2.12}$$

2.4.2 Performance estimation and cost structure

The probability of having a certain number of units in inventory, the outdate rate, and the probability of shortage and transshipment at each location can now be derived.

If $P_{j,i}$ is defined as the probability that there are j units in the stock at location i, by using the above results, $P_{j,i}$ for $j = 1, 2, \cdots, S_i$ can be obtained. If there are exactly j items in stock, there must be $S_i - j$ items on order; therefore, $\zeta^i_{S_i} \leq \zeta^i_{S_i-1} \leq \cdots \leq \zeta^i_{j+1} \leq L_i$ and $L_i \leq \zeta^i_j \leq \zeta^i_{j-1} \leq \cdots \leq \zeta^i_1 \leq m$. Then

$$P_{j,i} = \int_{L_i}^{m} \int_{L_i}^{x_1} \cdots \int_{L_i}^{x_j-1} \int_{0}^{L_i} \int_{0}^{x_j+1} \cdots \int_{0}^{x_{S_i-1}} C_i \cdot e^{-\int_0^{x_1} \bar{\lambda}_i dx} \, dx_{S_i} \cdots dx_2 \, dx_1 =$$

$$\frac{C_i.L_i^{S_i-j}}{(S_i-j)! \times (j-1)!} \times [\int_{L_i}^{k_i} e^{-\alpha_i x_1}.(x_1 - L_i)^{j-1} \, dx_1 + \int_{k_i}^{m} e^{-(\alpha_i k_i + \beta_i (x_1 - k_i))} . (x_1 - L_i)^{j-1} \, dx_1] \tag{2.13}$$

Similarly, the probability of stock-out can be obtained as follows:

$$P_{0,i} = \int_{0}^{L_i} \int_{0}^{x_1} \cdots \int_{0}^{x_{S_i-1}} C_i.e^{-\alpha_i L_i} \, dx_{S_i} \cdots dx_1 = \frac{C_i.e^{-\alpha_i L_i}.L_i^{S_i}}{S_i!} \tag{2.14}$$

using Equation (2.13), the expected number of items at location i is acquired as:

$$E(IL_i) = \sum_{j=1}^{S_i} j P_{j,i} \tag{2.15}$$

To obtain the probability of lateral transshipment at location i, a situation whereby an arriving customer is confronted with no stock at location i is considered. Then, if location j has stock on hand and the age of the oldest item in location j is greater than k_j, a lateral transshipment is triggered, and the transshipped item is allocated to the arriving customer. In contrast, if location j is out of stock or the age of the oldest item is less than k_j, an emergency order to the supplier (e.g., the blood bank) is requested. According to this discussion, the probability of lateral transshipment (LT in the equation) at location i and emergency order (EO in the equation) can be written as follows:

$$P(LT_i) = P_{0,i}.P_j(x_1 \geq k_j) \quad i, j \in \{1, 2\}, i \neq j \tag{2.16}$$

$$P(EO_i) = P_{0,i}(P_{0,j} + P_j(L_j \leq x_1 < k_j)) = P_{0,i}P_j(x_1 < k_j) \quad i,j \in \{1,2\}, i \neq j \quad (2.17)$$

where $P_{0,i}$ is the probability that location i is out of stock, $P_{0,j}$ is the probability that location j is out of stock and $P_j(x_1 \geq k_j)$ is the probability that the age of oldest item at location j is greater than or equal to k_j. Note that allowing emergency order is genuinely the lost-sale scenario. Finally, the outdate rate at location i would be $p_i(x_1 = m) \times (1 - p(d(\Delta) > 0)$, where $p(d(\Delta) > 0)$ is the probability of observing demand in time interval $\Delta \to 0$. Since demand is a Poisson process, $p(d(\Delta) > 0) = 0$, therefore, the outdate rate is obtained, as in (Schmidt and Nahmias (1985)):

$$O_i = C_i e^{-(\alpha_i k_i + \beta_i(m-k_i))}.\frac{m^{S_i-1}}{(S_i-1)!} \quad (2.18)$$

In order to compute the performance measure, the demand rate must be known. However, demand rate is a function of $P_{0,i}$. Therefore, a numerical procedure is developed to find the values of β_i in Equation (2.1). For finding the values of β_i, an iterative procedure is used, in which it is proved that β_i converges. β_i at iteration k is calculated by $\beta_i^k = \lambda_i + \lambda_j.P_{0,j}^k$, where $P_{0,j}^k$ is $P_{0,j}$ at k^{th} iteration (the superscript notation shows the iteration of the procedure). First, β_1^0 is calculated from Equation (2.1) for an initial value of $P_{0,2}^0 = 0$; then, $P_{0,1}^0$ is obtained from Equations (2.10) and (2.14). Inserting the value of $P_{0,1}^0$ in Equation (2.1) generates the value of β_2^0. Then $P_{0,2}^1$ is calculated form β_2^0 and β_1^1 is calculated using $P_{0,2}^1$. This procedure continues until no changes are observed in $\beta_i^k, i \in \{1,2\}$.

Proposition 2 (adapted from Proposition 2 in Olsson (2015)) The sequence $\{\beta_i^k, \quad k = 0,1,\cdots\}$ converges.

Proof. Based on the theorem of monotone convergence, the sequence $\{\beta_i^k, \quad k = 0,1,\cdots\}$ must be shown to be monotone and bounded. The sequence is bounded by $[\lambda_i, \lambda_i + \lambda_j]$. Further, the sequence $\{\beta_i^k, \quad k = 0,1,\cdots\}$ must be shown to be increasing.
As $\beta_i = \lambda_i + \lambda_j.P_{0,j}$, the following is clear:

(a) by increasing $P_{0,j}$, β_i will be increased as well

(b) an increase in β_i leads to higher $P_{0,i}$ (if β_i from Equation (2.9) is increased, C_i increases, which leads to an increase in $P_{0,i}$).

It can be demonstrated that the sequences $\{\beta_i^k = \lambda_i + \lambda_j.P_{0,j}^k, \quad k = 0, 1, \cdots\}$ will increase if $P_{0,j}^0 = 0$, which leads to $\beta_i^0 = \lambda_i$ as a starting condition. Inserting $\beta_i^0 = \lambda_i$ in Equation (2.10) generates the value C_i and $P_{0,i}^0 \geq 0$ is obtained from Equation (2.14). Next, the value $P_{0,i}^0$ in Equation (2.1) can be inserted to calculate β_j^0. Inserting this value in Equations (2.10) to (2.14) generates $P_{0,j}^1 \geq 0$, from which it is clear that $P_{0,j}^1 \geq P_{0,j}^0$. Therefore, $\beta_i^1 \geq \beta_i^0$ is obtained from (a), and according to (b), $P_{0,i}^1 \geq P_{0,i}^0$ and $\beta_j^1 \geq \beta_j^0$ is obtained from (a). The proof is completed by continuing this approach and induction over k to demonstrate that the sequence $\{\beta_i^k, \quad k = 0, 1, \cdots\}$ converges. □

As noted, the model captures the costs of inventory holding, emergency order, lateral transshipment, and outdate. Hence, the total expected cost in the case of allowing emergency orders (lost sale) would be:

$$EC = \sum_{i=1}^{2} \big(h_i E(IL_i) + \theta_i O_i + \rho_i \lambda_i P(LT_i) + q_i \lambda_i P(EO_i)\big) \tag{2.19}$$

The objective is to find $S_i's$ and k_i that minimize the expected total system cost. As the expected cost function is complex, it is not possible to prove that it is a convex function with respect to S_i. Therefore, the function is optimized numerically (see Section 2.6). In the unidirectional transshipment case that only location 1 transships item to location 2, the performance measures of location 2 are computed from the formulas presented in Schmidt and Nahmias (1985),which are presented in Section 2.6 (Equations (2.30) to (2.34)).

2.5 Inventory model for the designed lateral transshipment when backlogging is allowed

In this section, the proposed model is analyzed for the situation in which the demand that cannot be satisfied by the stock on hand or via lateral transshipment is backordered and it is not possible to initiate an emergency order. In the previous section, the state space had a finite dimension. When backorders are allowed the size of the state space grows to infinity. Hence, the previous solution method requires modification.

In the complete backlogging case, the number of items that can be in the system is not limited. However, we can consider the S_i youngest items without loss of generality because the other items are already assigned to customers and will satisfy their demands as soon as they arrive. Olsson and Tydesjö (2010) proved that whenever customers arrive, or a unit is outdated, non-zero lead time, the backorder system and zero lead time, the lost-sale systems trigger a replenishment order at the same time. Hence, the joint steady-state distribution of the age of the S_i items is similar in both systems.

To acquire the steady-state distribution of the process, a similar approach to that of Olsson and Tydesjö (2010) is used. Hence, according to (2.1) and (2.9), the joint density function of $x_1, \cdots, x_{S_i}$ is expressed as follows:

$$p_i(x_1, \cdots, x_{S_i}) = \begin{cases} C_i e^{-\alpha_i x_1}, & \text{if } x_1 < k_i\,, \\ C_i e^{-(\alpha_i k_i + \beta_i(x_1 - k_i))}, & \text{if } x_1 \geq k_i\,. \end{cases} \tag{2.20}$$

However, in the backorder case, Equation (2.1) needs some adjustments as $\alpha_i = \lambda_i$ and $\beta_i = \lambda_i + \lambda_j P(IL_j \leq 0)$, where $P(IL_j \leq 0)$ is the probability of having zero inventory at location j while there might be some backorders in the system (IL_j can be negative that shows the number of backorders can be negative, showing the number of backorders). $P_{0,i}$ in the backorder case is the probability that there is no stock on hand and no backorder. The constant C_i is obtained by integrating the density function over the entire domain. Hence,

$$\int_0^m \int_0^{x_1} \cdots \int_0^{x_{S_i-1}} C_i . e^{-\int_0^{x_1} \bar{\lambda}_i dx}\, dx_{S_i} \cdots dx_1 = 1$$

$$\rightarrow \int_0^m C_i . e^{-\int_0^{x_1} \bar{\lambda}_i dx} . \frac{x_1^{S_i-1}}{(S_i-1)!}\, dx_1 = 1$$

$$\rightarrow \frac{1}{C_i} = \int_0^{k_i} e^{-\alpha_i x_1} . \frac{x_1^{S_i-1}}{(S_i-1)!}\, dx_1 + \int_{k_i}^m e^{-(\alpha_i k_i + \beta_i(x_1 - k_i))} . \frac{x_1^{S_i-1}}{(S_i-1)!}\, dx_1 \tag{2.21}$$

The marginal distribution of the oldest item would be as follows:

$$p_i(x_1) = \begin{cases} C_i e^{-\alpha_i x_1} . \frac{x_1^{S_i-1}}{(S_i-1)!}, & \text{if } x_1 < k_i\,, \\ C_i e^{-(\alpha_i k_i + \beta_i(x_1 - k_i))} . \frac{x_1^{S_i-1}}{(S_i-1)!}, & \text{if } x_1 \geq k_i\,. \end{cases} \tag{2.22}$$

The performance measures are derived through the following process. First, considering $P_{j,i}$ the probability that there are j units ($j = 1, 2, ..., S_i$) in stock at location i, using Equations (2.20) and (2.21)gives rise to the following:

$$P_{j,i} = P(IL_i = j) = \int_{L_i}^{m}\int_{L_i}^{x_1}\cdots\int_{L_i}^{x_j-1}\int_{0}^{L_i}\int_{0}^{x_j+1}\cdots\int_{0}^{x_{S_i-1}} C_i{\cdot}e^{-\int_0^{x_1}\bar{\lambda}_i dx}\, dx_{S_i}\cdots dx_2\, dx_1 =$$

$$\frac{C_i.L_i^{S_i-j}}{(S_i-j)!\times(j-1)!}\times\int_{L_i}^{k_i}(x_1-L_i)^{j-1}.e^{-\alpha_i x_1}\,dx_1+\int_{k_i}^{m}(x_1-L_i)^{j-1}.e^{-(\alpha_i k_i+\beta_i(x_1-k_i))}\,dx_1 \tag{2.23}$$

The probability of there being no stock on hand at location i is given by:

$$P(IL_i \leq 0) = \int_0^{L_i} p_i(x_1)dx_1 = 1 - \sum_{j=1}^{S_i} P_{j,i} \tag{2.24}$$

As $k_i \geq L_i$ the probability density function of x_1 used in Equation (2.24) is the first expression in Equation (2.22) that is for domain of $x_1 < k_i$.

Because of backordering, the inventory can be negative (i.e., $\sum_{j=-\infty}^{S_i} P_{j,i} = 1$). The balance equation for moving from the state with negative inventory $-d$ is written according to Olsson and Tydesjö (2010):

$$\lambda_i P_j(x_1 \leq k_j)P_{(-d+1),i}+\frac{S_i+d+1}{L_i}P_{(-d-1),i} = \big(\lambda_i P_j(x_1 \leq k_j)+\frac{S_i+d}{L_i}\big)P_{-d,i} \qquad d = 1, 2, \ldots \tag{2.25}$$

The probability of having d units backordered determined by recursive calculation, as a function of $P_{0,i}$,

$$P_{-d,i} = \frac{S_i!}{(S_i+d)!}(\lambda_i P_j(x_1 \leq k_j)L_i)^d P_{0,i} \qquad d = 1, 2, \ldots \tag{2.26}$$

From Equations (2.24) and (2.26), $P_{0,i}$ was obtained as follows:

$$P_{0,i} = 1 - \sum_{j=1}^{S_i} P_{j,i} - \sum_{d=1}^{\infty} P_{-d,i} \tag{2.27}$$

The rate of outdating O_i in an approach similar to that explained for Equation (2.18) as in (Olsson and Tydesjö (2010)):

$$O_i = C_i.e^{-(\alpha_i k_i+\beta_i(m-k_i))}.\frac{m^{S_i-1}}{(S_i-1)!}. \tag{2.28}$$

Finally, the expected total system cost is as follows:

$$EC = \sum_{i=1}^{2} \big(h_i \sum_{j=1}^{S_i} j.P_{j,i} + \theta_i.O_i + b_i \sum_{k=-\infty}^{0} (-kP_{k,i}) \big) + \lambda_1\rho_1(1 - \sum_{j=1}^{S_1} P_{j,1})P_2(x_1 \geq k_2) +$$
$$\lambda_2\rho_2(1 - \sum_{j=1}^{S_2} P_{j,2})P_1(x_1 \geq k_1)\big) \quad (2.29)$$

In Equation (2.29), the first term is the expected holding cost, the second term is the expected outdate cost, the third term is the expected cost of backorders, and the fourth and fifth terms are the expected costs of transshipment. In the unidirectional-transshipment case, where only location 1 transships item to location 2, the fifth term in Equation (2.29) is removed and the performance measures of location 2 are calculated from the formulas provided in Olsson and Tydesjö (2010).

2.6 Numerical studies

In this section, the performance of the new model is illustrated and numerically evaluated through the iterative procedure proposed in Sections 2.4 and 2.5. To achieve this, five numerical experiments were carried out for the cases of lost sale and backlog.

To assess the effect of lateral transshipment on cost, the results of three sets of scenarios were compared. In the first scenario, lateral transshipment was used for both locations; in the second, only the location with lower demand could transship to the other location; and in the third, transshipment was not allowed in the case of stock-out (this scenario was based on the model of Schmidt and Nahmias (1985)).

When transshipment was not allowed, the performance measures different from those presented in Section 2.4 and were as presented below (Schmidt and Nahmias (1985)):

$$P_{0,i} = C_i.e^{-\lambda_i L_i}.\frac{L_i^{S_i}}{S_i!} \quad (2.30)$$

$$\frac{1}{C_i} = e^{-\lambda_i L_i}.\frac{L_i^{S_i}}{S_i!} + \int_{L_i}^{m} e^{-\lambda_i x_1}.\frac{x_1^{S_i-1}}{(S_i-1)!}\,dx_1 \quad (2.31)$$

$$P_{j,i} = \frac{C_i L_i^{S_i - j}}{(S_i - j)! \times (j-1)!} \Big(\int_{L_i}^{m} e^{-\lambda_i x_1} . (x_1 - L_i)^{j-1} \, dx_1 \Big) \tag{2.32}$$

$$O_i = \frac{C_i e^{\lambda_i m} m^{S_i - 1}}{(S_i - 1)} \tag{2.33}$$

$$E(IL_i) = \sum_{j=1}^{S_i} j P_{j,i} \tag{2.34}$$

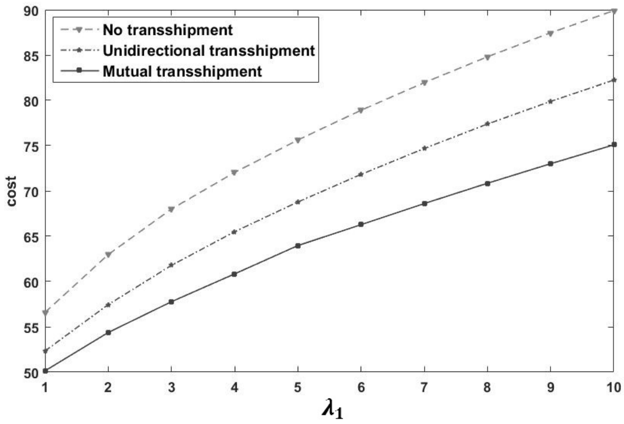
$$EC = \sum_{i=1}^{2} \big(h_i . E(IL_i) + \theta_i . O_i + q_i . \lambda_i . P_{0,i} \big) \tag{2.35}$$

In the case of unidirectional transshipment, only the location with lower demand rate was allowed to transship when the other location was out of stock. The performance measures for the location with lower demand were as in Equations (2.15) to (2.19), and for the location with higher demand were calculated from Equations (2.30) to (2.35).

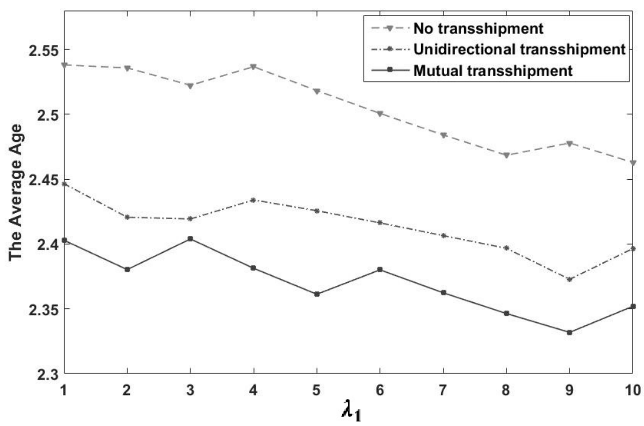
The lateral-transshipment cost was set to be \$8 per shipment ($\rho_i = 8$) and the emergency-order cost was equal to \$15 per order ($q_i = 15$). The holding cost was $h_i = 7$ per unit per time; the outdate cost was $\theta_i = 10$ per unit; and the shelf life was $m = 8$.

The first example considered $\lambda_2 = 10$ and increased λ_1 from 1 to 10. Although from a modeling perspective, it was not necessary that $L1 = L2$, $L1 = L2 = 2$ was considered. It is reasonable to consider $L1 = L2$ because it is vital of importance that two locations are geographically close to each other to are allowed to transship together. Figure 2.1a demonstrates how changing λ_1 affected the total cost. Figure 2.1b shows that as λ_1 increased, the total cost of the system increased as well, and this behavior was observed in all three models. The results demonstrated that the system had the lowest total cost in the mutual-transshipment case and the largest total cost would occur when transshipment was not allowed. Further, the cost differences were more noteworthy as the demand rate at location 1 was increased. Table 2.1 represents detailed results including the optimal order-up levels, shortage cost and holding cost related to Figure 2.1a. The results highlighted that as λ_1 increased all $S_1^\star$, the holding and shortage costs increased as well. Other parameters and performance measures for complete comparison are provided in Appendix A.

One of the key performance metrics for a system with perishable items, particularly



(a) The total cost of system.



(b) The average age of the issued items.

FIGURE 2.1: The total cost and the average age of issues for various transshipment scenarios ($m = 8$, $\lambda_2 = 10$ and λ_1 varies from 1 to 10).

blood, is the age of the issued items. Recent findings have suggested that transfusing older blood may cause some difficulties, particularly for critically ill patients. Wang et al. (2012) demonstrated that utilizing older stored blood resulted in a significant increase in the risk of death. Therefore, the total average age of items in three scenarios of transshipment were compared. The demand rate of location 2 was considered constant and was equal to 10, and again, the demand rate of location 1 was varied from 1 to 10. The average age of issued items from location i (denoted by $E(A_i)$) was the average of the age of the oldest item (ζ_1^i) in location i and was obtained by:

$$E(A_i) = E(\zeta_1^i | \zeta_1^i \geq L_i) = \int_{L_i}^{m} x_1 p_i(x_1 | x_1 \geq L_i) dx_1. \tag{2.36}$$

Then, the average age of issued items in the system was computed as the weighted average of age of issued items at both locations, approximated as:

$$E(A) \simeq \frac{\lambda_1}{\lambda_1 + \lambda_2} E(A_1) + \frac{\lambda_2}{\lambda_1 + \lambda_2} E(A_2) \tag{2.37}$$

The results presented in Figure 2.1b indicated that mutual transshipment dominates the other models and issues slightly fresher items. In addition, compared to unidirectional transshipment, mutual transshipment further improved the average age of transfused units. This is because the transshipped items were used immediately after transshipment (as the defined transshipment policy was reactive and transshipment occurred when a location faced shortages), whereas in the no-transshipment policy, those items could stay longer in the inventory. In some of the cases, when transshipment was applied, the order-up levels (S_i's) increased (e.g., rows 3 to 10 in Table 2.1) and the holding cost and the average age of issued items still improved. This was because of the increased demand rates in the system with transshipment (i.e., the adjusted demand rate was higher than original demand rate – refer to Equation (2.1)).

Table 2.1 shows that in all cases, the mutual-transshipment scenario performed significantly better than the unidirectional-transshipment and no-transshipment scenarios. Interestingly, the total stock in the case of no transshipment was lower than in the two other scenarios but the total holding cost in the case of no transshipment was higher than it was in the other scenarios. This was because of the longer stay of the items in inventories, which increased the age of items issued.

To study the effect of lead time and transshipment on costs, $\lambda_1 = 5$, $\lambda_2 = 10$, $m = 8$ were set and the lead time varied from 0.5 to 3. The cost parameters were same as in the previous example. The results of this experiment are presented in Table 2.2. As expected, as the lead time increased, the total cost of the system also increased. Table 2.2 shows that the total cost obtained when using mutual transshipment was always lower than it was in the two other scenarios; and the cost difference between the unidirectional-transshipment scenario and the no-transshipment scenario increased as the lead time increased.

In the next experiment, the effect of the transshipment cost on the total cost of the system was investigated, considering $\lambda_1 = 8$, $\lambda_2 = 10$, $h_i = 7$, $\theta_i = 20$, $q_i = 15$, $m = 8$ and $L = 2$ and changing ρ from 8 to 14. In the case of no transshipment, the total cost of the system was 84.810. The results of this experiment in the cases of unidirectional transshipment and mutual transshipment are presented in Table 2.3. Table 2.3 reveals the minimal cost of the system, as well as the related optimal base-stock level for the unidirectional-transshipment policy and the mutual-transshipment policy for both locations. It was expected that in both transshipment policies, the total cost of system would increase as the transshipment cost increased. This is verified in Table 2.3. The results presented in Table 2.1 indicate that in all cases, a mutual-transshipment policy performed significantly better than a unidirectional-transshipment policy. When the transshipment cost increased, the difference between the total costs of the unidirectional-transshipment policy and the mutual-transshipment policy decreased. That is, mutual transshipment was most beneficial when the transshipment cost was lower.

The two types of decision variables in the system ((i.e., the order-up level (S_i) and the age threshold (k_i)) were connected and decided simultaneously. In addition, because of the complexity of the problem, the optimal solutions of $S_i's$ and $k_i's$ were obtained numerically. Therefore, it was not possible to mathematically investigate the effect on the decision variables of changing parameters such as lead time, unit transshipment cost, demand rate, and shelf life. However, the numerical studies showed that by increasing the demand rate from 1 to 10, while S_i increased, k_i did not change. Therefore, k_i was not highly sensitive to demand rates (see Table A.1). The results in Table A.2 show that by increasing demand rates when S_i increased, k_i tended to decrease, and vice versa. However, when lead time increased both S_i and k_i increased (see Table 2.2) and when unit transshipment cost increased k_i tended to increase, while S_i might decrease or increase (see Table 2.3). In addition, numerical studies were performed by changing m, and observed that k_i was not sensitive to m. This was because even for small m, the outdate rate was negligible because of the Poisson demand distribution.

	No Transshipment				**Unidirectional Transshipment**				**Mutual Transshipment**			
	S_1^*	S_2^*	Hold.C	Short.C	S_1^*	S_2^*	Hold.C	Short.C	S_1^*	S_2^*	Hold.C	Short.C
$\lambda_1 = 1$	2	22	34.512	22.037	4	21	32.918	12.770	4	21	30.262	9.9241
$\lambda_1 = 2$	4	22	37.640	25.332	7	20	34.054	13.986	6	21	30.707	10.401
$\lambda_1 = 3$	6	22	40.069	27.932	9	20	35.982	16.053	8	22	35.535	8.073
$\lambda_1 = 4$	9	22	45.639	26.398	12	20	40.985	13.913	10	22	36.008	8.580
$\lambda_1 = 5$	11	22	47.369	28.252	14	20	42.515	15.552	12	22	36.488	9.094
$\lambda_1 = 6$	13	22	48.954	29.951	16	20	43.941	17.080	15	22	41.530	6.996
$\lambda_1 = 7$	15	22	50.426	31.528	18	20	45.283	18.517	17	22	41.981	7.480
$\lambda_1 = 8$	17	22	51.805	33.005	20	20	46.553	19.878	19	22	42.430	7.9603
$\lambda_1 = 9$	20	22	56.704	30.754	23	19	46.644	19.975	21	22	42.875	8.438
$\lambda_1 = 10$	22	22	57.886	32.020	25	20	52.323	18.561	23	23	48.194	6.637

TABLE 2.1: The shortage and the holding costs of system with $m = 8$, $L_1 = L_2 = 2$ $\lambda_2 = 1$ and λ_1 alters from 1 to 10 (lost-sale case). Further results on this numerical study are presented in Table A.1 in Appendix A.

No Transshipment				**Unidirectional Transshipment**				**Mutual Transshipment**				
	S_1^*	S_2^*	EC	S_1^*	S_2^*	k_1^*	EC	S_1^*	S_2^*	k_1^*	k_2^*	EC
$L = 0.5$	5	8	57.909	5	8	2	53.720	4	8	2	2	50.326
$L = 1.0$	7	13	66.812	8	12	2	63.577	7	13	2	2	57.1736
$L = 1.5$	9	17	72.147	11	16	2	66.571	10	17	2	2	61.138
$L = 2.0$	11	22	75.621	14	20	2	68.778	12	22	2	2	63.966
$L = 2.5$	13	26	78.160	17	23	2	71.151	15	26	2	2	65.525
$L = 3.0$	15	30	80.142	20	27	3	72.576	18	30	3	3	66.951

TABLE 2.2: The total cost of system for different lead times ($\lambda_1 = 5$, $\lambda_2 = 10$, $L_1 = L_2 = L$ and $m = 8$)(lost-sale case).

	Unidirectional Transshipment				**Mutual Transshipment**				
	S_1^*	S_2^*	k_1^*	EC	S_1^*	S_2^*	k_1^*	k_2^*	EC
$\rho = 8$	20	20	2	77.386	19	20	2	2	70.847
$\rho = 9$	20	20	2	78.755	19	22	2	2	73.404
$\rho = 10$	20	20	2	80.125	19	23	2	2	75.714
$\rho = 11$	20	21	2	81.300	19	23	2	2	77.958
$\rho = 12$	20	21	2	82.451	19	23	2	2	80.201
$\rho = 13$	20	21	3	83.601	19	24	3	3	82.432
$\rho = 14$	18	22	3	84.249	18	22	3	3	83.844

Table 2.3: The total cost in different transshipment polices ($\lambda_1 = 8$, $\lambda_2 = 10$, $m = 8$ and $L_1 = L_2 = 2$)(lost-sale case).

	No Transshipment			**Unidirectional Transshipment**			**Mutual Transshipment**		
	S_1^*	S_2^*	EC	S_1^*	S_2^*	EC	S_1^*	S_2^*	EC
$\lambda_1 = 1$	3	22	55.601	3	43	28.445	16	16	16.953
$\lambda_1 = 2$	5	22	60.100	9	39	32.943	18	16	17.769
$\lambda_1 = 3$	8	22	63.686	8	33	36.424	19	17	18.547
$\lambda_1 = 4$	10	22	66.407	10	34	39.204	20	18	19.414
$\lambda_1 = 5$	12	22	68.895	12	37	41.730	24	17	20.232
$\lambda_1 = 6$	14	22	71.202	14	35	44.036	25	18	21.420
$\lambda_1 = 7$	16	22	73.363	16	37	46.200	25	20	23.390
$\lambda_1 = 8$	16	22	75.403	19	28	48.241	25	22	24.659
$\lambda_1 = 9$	20	22	77.339	21	28	50.064	25	24	25.403
$\lambda_1 = 10$	22	22	79.187	23	28	51.813	27	25	27.127

Table 2.4: The total costs of system for λ_1 from 1 to 10 in backorder case.

Table 2.4 presents the evaluation of the performance of transshipment policies when backlogging was allowed. This used the same model as that used in Olsson and Tydesjö (2010) with transshipment not allowed in a case of stock-out. Similar results to the lost-sale scenario were expected. The parameters for this example were $b_i = 15$, $\rho_i = 8$, $\theta_i = 10$ and $h_i = 7$. The results demonstrate that mutual transshipment is more advantageous and the cost difference between the no-transshipment scenario and the transshipment scenarios are significant. Note that the performance of unidirectional and mutual transshipment depend on the performance metrics measured and the transshipment cost per unit. However, the mutual-transshipment policy always dominated the unidirectional-transshipment policy, as in the mutual-transshipment policy $k_2 = m$ converted the mutual transshipment to the unidirectional transshipment.

2.6.1 Comparison with current practice in some hospitals

In the final experiment, this new transshipment policy was evaluated and compared against the transshipment policy currently practiced in some hospitals in Australia, in which small hospitals can proactively transship their units of red blood cells to large hospitals when these units have a residual shelf life of less than 14 days. This experiment postulated two hospitals (one large and one small), with demand rates of 13 and 2.5 units per day, respectively. In Australia, the shelf life of a red blood cell is 42 days from the date of donation and the average age of issue of red blood cells to a hospital is 9.8 days (Abbasi, Vakili, and Chesneau (2017)); therefore, the shelf life of items after arriving at hospitals was assumed at approximately 32 days. All of the cost components at the two hospitals were equal. For both hospitals, the shortage cost was three times higher than the holding cost, the outdate cost was four times higher than the holding cost, and the transshipment cost was 2.5 times higher than the holding cost.

The results of this model are presented in Table 2.5 which shows the optimal total cost, the average age of issues and the rate of outdate obtained from the mutual-transshipment and the unidirectional-transshipment policies. These results were then compared with the results obtained by simulating the transshipment policy currently used in practice in some Australian hospitals. The average age of issued blood and the outdate rate of the current transshipment policy were 12.60 and 0.00035, respectively.

Table 2.5 demonstrates that the new transshipment policy outperformed the current transshipment policy, and the unidirectional-transshipment policy performed as well as the mutual-transshipment policy. The results demonstrated that compared with the current policy, the new policy resulted in the same average age of issued items while reducing the total cost. Despite the fact that this transshipment policy was easy to implement, the tests illustrated that utilizing this approach could be significantly useful from a cost perspective.

Current transshipment			**Unidirectional transshipment**				**Mutual transshipment**			
EC	$E(A)$	$O = O_1 + O_2$	k_1^*	EC	$E(A)$	$O = O_1 + O_2$	(k_1^*, k_2^*)	EC	$E(A)$	$O = O_1 + O_2$
29.80	12.60	0.00035	13	11.54	12.59	0.000003	(13,14)	11.51	12.55	0.000005

TABLE 2.5: The performance measures of the system.

2.7 Summary

In this chapter, a new approach for modeling reactive transshipment in the context of perishable-inventory systems has been proposed. This approach has been utilized to determine stock levels while considering strategies for transshipment. Based on a partial differential equation, the joint distribution of the age has been derived and used to determine the costs and to optimize the decision variables (i.e., the optimal inventory level at each location and the transshipment policy that was based on the age of the items in stock). The results revealed that the simple decision rules that are practiced in some blood supply chains, such as always requesting emergency orders from the supplier in the case of stock-out, or transshipping items within an agreed number of days from stock expiry to the larger location (i.e., the location with the higher rate of demand), are far from optimal. Through various numerical results, this new transshipment policy has indicated significant cost savings. In addition, this policy would slightly ameliorate the average age of transfused items, which is an important performance measure in the blood supply chain.

Chapter 3

Blood transshipment in a hospital network

3.1 Introduction

In this chapter, proactive transshipment is applied as a powerful mechanism to rebalance the blood inventory in a network of hospitals as a way of diminishing the mismatch between demand and supply and reducing the total cost of the holding, outdate, shortage, and transshipment costs. A dynamic programming model is developed to analyze the effect of proactive transshipment on the blood supply chain. A system with a network of hospitals (one main hospital and some small hospitals) is considered, with all facing uncertain demand, with a general probability distribution. At the end of each period, each hospital has the option to replenish its inventory from the blood bank to meet the demand of the next period. Moreover, small hospitals can transship blood products to the large hospital at the end of each period.

The dynamic programming formulation of the problem studied in this research suffers heavily from the curse of dimensionality because of the massive size of the state and decision spaces. To overcome the curse of dimensionality, one of the approximate dynamic programming (ADP) methods from the literature is applied. Specifically, the value function of the dynamic programming is approximated with a set of basis functions, with different weights assigned to these functions by solving a linear program model using a column generation algorithm. Subsequently, this approximate value function is deployed to determine the approximate optimal order quantity and the approximate optimal transshipment quantity in each period by solving an integer program.

The remainder of this chapter is organized as follows. Section 3.2 presents a review of the related literature on blood inventory management and proactive transshipment. Section 3.3 contains a detailed description of the proposed model and the mathematical formulation of the model. Section 3.4 presents the ADP method used to solve the proposed model. Section 3.5 contains a numerical study and Section 3.6 summarizes this chapter.

3.2 Literature review

The literature review related to this part of the study is presented in two categories: blood inventory management and proactive transshipment.

3.2.1 Blood inventory management

Potentially, most of the perishable-inventory theory can be applied to blood inventory management. However, the literature on blood inventory management is rather limited. Effective techniques (e.g., just-in-time) that are applied in industrial and commercial business sectors to manage inventory do not adjust well to the blood supply chain, because of inventory shortage (Chapman, Hyam, and Hick (2004)).

The research related to blood inventory management was initiated in the early 1960s by Millard (1959), Zyl (1963), and Silver and Silver (1964). The literature can be categorized into two major periods of activity: the 1970s, when regression models were proposed for the first time to determine optimal order quantities and inventory levels; and the 2000s, which was dominated by simulation and advanced operations research techniques.

Jennings (1973) analyzed whole blood inventory control policies at hospitals and identified shortage, outdating/ wastage, and the cost of information and transportation as three key measures of performance.

Brodheim, Derman, and Prastacos (1975) applied a finite-state Markov chain and statistical regression techniques to develop an inventory model for blood. From its stationary distribution, they derived an expression relating to shortage rate, the average

age of the inventory, and the average wastage per period. Cumming et al. (1976) used a Markovian population model to develop a planning model for blood collection and to assist regional blood suppliers to deal with seasonal imbalances between supply and demand. Subsequently, Cohen and Pierskalla (1979) developed decision rules that specified optimal target inventory levels, considering average daily demand and the average transfusion-to-crossmatch ratio as parameters, using a linear regression model.

Prastacos and Brodheim (1980) developed a mathematical model to optimize the allocation of blood in a centralized system, which was implemented in the 38 hospital regions of Long Island, New York. Cohen and Pierskalla (1979) developed a simple equation model to optimize hospitals' target stock levels. They considered factors such as demand rates, the transfusion-to-crossmatch ratio, and the crossmatch release time. In a later article, Prastacos (1984) provided a review of research in blood inventory management for the first time. Then, nearly 20 years later Owens, Tokessy, and Rock (2001) studied the effect of the shelf life of blood units on the inventory performance. They found that increasing the shelf life could yield considerable reductions in wastage.

The establishment of the Blood Stocks Management Scheme in England and North Wales to review performance in terms of stock levels and blood wastage (Chapman and Cook (2002)) increased new research opportunities in blood inventory management. In addition, advances in computer technology improved the availability of simulation tools to develop meaningful models of this complicated process. Various simulation techniques have been used to optimize the blood supply chain, despite the fact that the optimality of the solution obtained could not be guaranteed. Examples of applying simulation techniques to the blood supply chain can be found in Rytilä and Spens (2006), Duan and Liao (2013), Mustafee et al. (2009), Kamp et al. (2010), Abbasi, Vakili, and Chesneau (2017), and Blake et al. (2013b).

A number of techniques have been used to analyze the supply chain of blood products. The most commonly applied methods are: queuing theory and Markov chains (Pegels and Jelmert (1970), Abbasi and Hosseinifard (2014), Brodheim, Derman, and Prastacos (1975), Cumming et al. (1976), Kopach, Balcıoğlu, and Carter (2008), and Hosseinifard and Abbasi (2018)); Statistical analysis, such as linear regression, survival analysis, and logistic regression (Bosnes, Aldrin, and Heier (2005), Godin et al. (2007), Heddle et al. (2009), and Perera et al. (2009)); and optimization methods such as integer programming, linear programming, and two-stage stochastic programming (Pitocco and Sexton (2005), Hemmelmayr et al. (2009), Şahin, Süral, and Meral (2007), Gunpinar and Centeno (2015), Fattahi et al. (2015), and Dillon, Oliveira, and Abbasi

(2017)).

In addition, dynamic programming models have been applied to analyze the blood supply chain, but less frequently because of the curse of dimensionality. Some methods, such as ADP, can be applied to this problem. Dynamic programming models have been used to analyze blood inventory by Haijema, Wal, and Dijk (2007), Van Dijk et al. (2009), Blake (2009), Blake et al. (2003), Abdulwahab and Wahab (2014), and Zhou, Leung, and Pierskalla (2011).

Haijema, Wal, and Dijk (2007) applied a combination of stochastic dynamic programming and simulation approaches to the data from a Dutch blood bank to find the "near optimal" inventory policies for platelet production.

Van Dijk et al. (2009) applied stochastic dynamic programming to the blood platelet inventory management. They combined simulation with stochastic dynamic programming and proposed a five-step procedure to solve the problem for a real case study.

Haijema, Wal, and Dijk (2007) and Van Dijk et al. (2009) assumed that the age distribution of platelet production could not affect optimal ordering decisions, while Blake (2009) showed that it could not always be ignored when placing an order. The author mentioned that in spite of the potential for using a dynamic programming approach to optimize perishable-inventory policies, even for a restricted problem environment the curse of dimensionality prevented the implementation of their model.

Blake et al. (2003) adopted a dynamic programming method to solve a blood platelet inventory problem. They aggregated orders and demand into larger units, to make the problem tractable and to overcome the curse of dimensionality.

Abdulwahab and Wahab (2014) applied a set of methodologies, such as the newsvendor problem and ADP, to formulate the inventory of blood platelets, considering eight blood types with stochastic demand and stochastic supply.

Zhou, Leung, and Pierskalla (2011) used a dynamic programming model to analyze a blood platelet problem with three days of shelf life and stochastic demand, incorporating two modes of replenishment: every other day, or emergency order in between if necessary.

3.2.2 Proactive transshipment

Transshipment has been considered in the literature as a tool for balancing inventory among locations in the same echelon, to reduce shortage. Lateral-transshipment policies can be classified into proactive and reactive transshipment (Paterson et al. (2011)). Most past studies have considered reactive transshipment, in which transshipment occurs when an inventory shortage is realized (Axsäter (1990), Banerjee, Burton, and Banerjee (2003), Burton and Banerjee (2005), Herer, Tzur, and Yücesan (2006), Liang et al. (2014), Park, Lai, and Seshadri (2016), Tang and Yan (2010), Wee and Dada (2005), Yang and Qin (2007), Yao, Zhou, and Zhuang (2016), Zhang (2005), and Zhao and Atkins (2009)). In these studies, the transshipment time was considered negligible, to make the problem tractable.

Proactive transshipment takes place at fixed points in time, before observing a demand. Most of the work on proactive transshipment has considered a periodic-review setting. Allen (1958) presented a multi-echelon redistribution model for proactive transshipment for the first time. The author considered a single period with multiple inventory locations and obtained an optimal redistribution of stock. Agrawal, Chao, and Seshadri (2004) proposed a dynamic programming formulation in which the transshipment time was determined dynamically. They developed an algorithm to obtain the optimal time of the transshipment and stock levels at retailers. Lee, Jung, and Jeon (2007) proposed a new proactive transshipment policy, called the Service Level Adjustment policy, in which they considered the service level to determine the quantity of transshipment. Burton and Banerjee (2005) used simulation to analyze and compare the cost effects of proactive transshipment and reactive transshipment in a two-echelon supply chain network.

Tagaras and Vlachos (2002) considered a two-location system with non-negligible transshipment times and used simulation to analyze the operational characteristics of a pooling policy. They found that proactive transshipment was beneficial, especially when the demand was highly variable. Jönsson and Silver (1987) investigated a two-echelon distribution system with a central warehouse. They limited the timing of transshipment to minimize the total backorders. Lee and Whang (2002) considered a two-period model consisting of a manufacturer and several retailers, and obtained optimal stock level as well as optimal proactive transshipment policy. Rong, Snyder, and Sun (2010) proposed a proactive transshipment policy in a decentralized system with two stores. They considered two demand subperiods in which replenishment orders were made before the first subperiod and between the subperiods, the stores were allowed to transship to each other.

Li, Sun, and Gao (2013) obtained the optimal quantity of orders, taking into account proactive transshipment for a decentralized system with two locations. Glazebrook et al. (2015) proposed a hybrid lateral-transshipment policy, such that the transshipment decisions were made when a location faced a shortage that resembled a reactive transshipment policy. However, the quantity of transshipment could exceed the current shortage, to avoid a future imbalance in the inventory system. Glazebrook et al. (2015) used dynamic programming to solve their model, and a heuristic to approximate the future cost of a decision. In approximating this future cost, they assumed no transshipment was performed in the future.

Recently, Abouee-Mehrizi, Berman, and Sharma (2015) proposed a proactive transshipment model to minimize the mismatch between supply and demand. They considered a finite-horizon multi-period inventory system for two locations and determined optimal joint-replenishment and transshipment policies. Meissner and Senicheva (2017) considered a multi-location, multi-period inventory system with proactive transshipment and used ADP to determine an optimal order policy and transshipment policy.

The above studies analyzed the effect of lateral transshipment on the performance of a non-perishable inventory. Dehghani and Abbasi (2018) proposed a model for lateral transshipment of perishable products. Their model was limited to a Poisson demand distribution and worked only for the transshipment of perishable items between two locations. It is likely that the new model presented in this chapter is the first to analyze the effect of proactive transshipment on blood supply chain using ADP.

3.3 The hospital network setting and notations

The definitions and notations used to develop the model are introduced as follows:

Indices and sets

$\mathcal{N}$ - The set of hospitals. The hospital with index 1 is the large (main) hospital in the network and there are $N-1$ small hospitals.
M - The maximum shelf life.

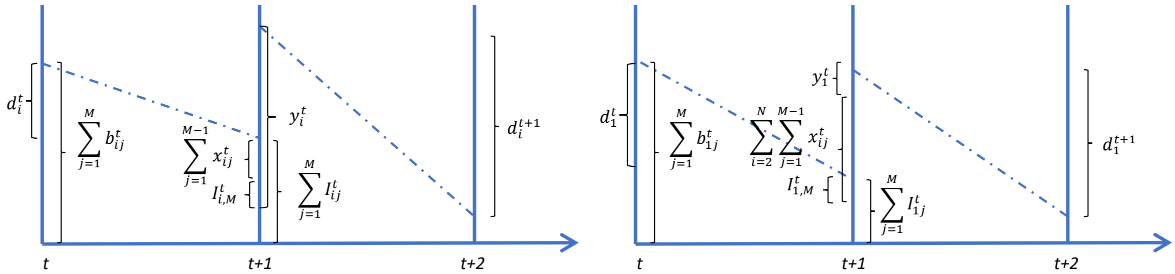
t - The index for the period. $t+1$ refers to the next period.

Decision variables

y_i^t - The order quantity of hospital i in period t. It is a decision variable. The notation without t refers to the current period order.
$x_{i,j}^t$ - The transshipment quantity of units with age $j \in \{1, 2, ..., M-1\}$ from hospital $i, i \in \mathcal{N} \setminus \{1\}$ to the main hospital in period t. It is a decision variable. The notation without t refers to the current period transshipment.
$I_{i,j}^t$ - Total inventory with age j at hospital $i \in \mathcal{N}$ at the end of period t after fulfilling the demand of period t (and before updating based on transshipment and placed order in period t). If t is dropped from the notation, $I_{i,j}$ and $I_{i,j}^{'}$ refer to $I_{i,j}^t$ and $I_{i,j}^{t+1}$ respectively. $I_i^t = (I_{i,1}^t, I_{i,2}^t, ..., I_{i,M}^t)$.
F_i^t - The shortage at hospital $i \in \mathcal{N}$ at the period t.
$a_{i,j}^t$ - The fulfilled demand from $I_{i,j}^t$ at period t at hospital i.

Parameters

o_i - The outdate cost per unit in the hospital $i, i \in \mathcal{N}$.
h_i - The holding cost per unit of time at hospital i.
q_i - The transshipment cost per unit for transshipping a unit from hospital $i, i \in \mathcal{N} \setminus \{1\}$ to hospital 1 .
g_i - The shortage cost per unit at hospital i. It is the equivalent of emergency-order cost per unit from the CBB.
c_i - The order cost per unit.
$b_{i,j}^t$ - Total inventory with age j at hospital $i \in \mathcal{N}$ at the beginning of period t before fulfilling the demand of period t.
D_i - The demand at hospital $i \in \mathcal{N}$ as a random variable. The probability mass function of D_i is denoted by $p(D_i)$. $D = (D_1, ..., D_N)$: $\mathcal{D}$ (as a set) is the support of D and $\mathcal{D}_i$ is the support of D_i
d_i^t - The demand at hospital i at period t. d_i denotes that the demand is referred to the current period that is known and $d_i^{'}$ denotes the demand in the next period.
S^t - The state of the system at time t. $S^t = (I_1^t, I_2^t, ..., I_N^t)$ where $I_i^t = (I_{i,1}^t, I_{i,2}^t, ..., I_{i,M}^t)$ for $i \in \mathcal{N}$.



(a) a small hospital (b) the main hospital

FIGURE 3.1: Notion of inventory over two periods in a small and the large (main) hospital that are linked together for transshipment purpose.

In the setting for this research, a large (main) hospital is denoted by 1 and $N-1$ small hospitals are indexed as $i = 2, 3, ..., N$. The demand of the current period is known to hospitals. The hospitals place orders $(y_i, i \in \mathcal{N})$ to the CBB at the beginning of each period (day) and these are received at the hospitals at the end of the period (day) and cannot be used in the current period. The units received from the central blood bank (CBB) have a shelf life of M and can be used in the next M periods, after which they will be outdated. In addition, at the beginning of each period, small hospitals may transship some units to the main hospital; these are available at the main hospital at the end of current period and can be used in the next period. This setting is based on the inventory and transshipment setting explained in Abbasi, Vakili, and Chesneau (2017).

Figures 3.1a and 3.1b illustrate the change (notion) of inventory in the time horizon. In Figures 3.1a and 3.1b, d_i^t and d_1^t can be replaced by $\sum_{j=1}^{M} a_{i,j}^t$ and $\sum_{j=1}^{M} a_{1,j}^t$, respectively. As the shortage per unit cost is the same in all periods, the model will not tolerate a shortage while having the inventory to fulfill the demand in the current period, which maintains the inventory to avoid a shortage in future.

At the beginning of each period, the inventory status and the demand of the current period are known. In case of a shortage at the hospital, the demand is fulfilled by placing an emergency order (with the cost of g_i per unit for hospital $i \in \mathcal{N}$) to the CBB and the order is received immediately. In this model, the emergency order cost is considered the shortage cost. Therefore, the decisions on orders and transshipment for each period can be made at the beginning of that period. In the following section, the problem is formulated by using a dynamic programming framework.

3.4 Dynamic programming formulation and solution approach

The decisions on orders and transshipment in the current period depend on the inventory positions at each hospital in the current period. Therefore, in the dynamic programming formulation, the state of the system is defined according to the inventory status. The state of the system at the end of period t is denoted by $S^t = (I_1^t, I_2^t, ..., I_N^t)$ where $I_i^t = (I_{i,1}^t, I_{i,2}^t, ..., I_{i,M}^t)$. The vector of possible actions is defined as χ^t. At the end of each period (e.g., t), after fulfilling the current demand, the new orders (y_i^t) arrive at the hospitals, as well as transshipped items (x_{ij}^t) from the small hospitals to the main hospital. Once a decision is made, the next period's demand is stochastic. Thus, if demand is represented by D^{t+1}, then the state transition, $(I_1^t, I_2^t, \cdots, I_N^t) \longrightarrow (I_1^{t+1}, I_2^{t+1}, , \cdots, I_N^{t+1})$ occurs with probability $P(D^{t+1} = (d_1^{t+1}, ..., d_N^{t+1}))$. Therefore,

$$p(S^{t+1}|S^t, \chi^t) = \begin{cases} P(D^{t+1} = (d_1^{t+1}, ..., d_N^{t+1})), & \text{if } S^{t+1} = (I_1^{t+1}, \cdots, I_N^{t+1}) \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

where D^{t+1} is the random demand vector $D = (D_1^{t+1}, \cdots, D_N^{t+1})$ and the element of new state is obtained as:

$$\begin{aligned} I_{i,j}^{t+1} &= I_{i,j-1}^t - a_{i,j}^{t+1} - x_{i,j-1}^t, \; i \in \mathcal{N} \setminus \{1\}, j = \{2, ..., M\}; \\ I_{i,1}^{t+1} &= y_i^t - a_{i,1}^{t+1}, \; i \in \mathcal{N}; \\ I_{1,j}^{t+1} &= I_{1,j-1}^t + \sum_{i \in \mathcal{N} \setminus \{1\}} x_{i,j-1}^t - a_{1,j}^{t+1}, j = \{2, ..., M\}; \end{aligned} \tag{3.2}$$

where $a_{i,j}^{t+1}$ is defined as follows:

$$
\begin{aligned}
& d_i^{t+1} = \sum_{j=1}^{M} a_{i,j}^{t+1} + F_i^{t+1},\ i \in \mathcal{N}; \\
& 0 \leq a_{i,j}^{t+1} \leq I_{i,j-1}^{t} - x_{i,j-1}^{t},\ i \in \mathcal{N} \setminus \{1\}, j = \{2, ..., M\}; \\
& 0 \leq a_{1,j}^{t+1} \leq I_{1,j-1}^{t} + \sum_{i=2}^{N} x_{i,j-1}^{t}, j = \{2, ..., M\}; \\
& 0 \leq a_{i,1}^{t+1} \leq y_i^t,\ i \in \mathcal{N}; \\
& 0 \leq F_i^{t+1},\ i \in \mathcal{N};
\end{aligned}
\tag{3.3}
$$

This model defines the immediate cost (which is function of the current state and action, and associated with satisfying a given period's demand) of implementing action χ^t when the system is in state S^t (therefore, both inventory at the beginning of the period ($b_{i,j}^t$) and the demand of the current period are known) as follows:

$$
C^t(S^t, \chi^t) = \sum_{i \in \mathcal{N}} (c_i y_i^t + o_i\, I_{i,M}^t + h_i\, (y_i^t + \sum_{j=1}^{M-1} I_{i,j}^t) + g_i \mathbb{E}(F_i^{t+1})) + \sum_{i \in \mathcal{N} \setminus \{1\}} \sum_{j=1}^{M-1} q_i^t x_{ij}^t \tag{3.4}
$$

where $\mathbb{E}(F_i^{t+1})$ is obtained from the following equations:

$$
\begin{aligned}
\mathbb{E}(F_i^{t+1}) &= \sum_{d_i^{t+1} \in \mathcal{D}_i} (d_i^{t+1} - \sum_{j=1}^{M} a_{i,j}^{t+1}) p(D_i^{t+1} = d_i^{t+1}) \\
&= \sum_{d_i^{t+1} \geq \sum_{j=1}^{M} b_{i,j}^{t+1}} (d_i^{t+1} - \sum_{j=1}^{M} b_{i,j}^{t+1}) p(D_i^{t+1} = d_i^{t+1}),\ i \in \mathcal{N};
\end{aligned}
\tag{3.5}
$$

χ^t includes the orders made by each hospital, the transshipment from small hospitals to the main hospital, and the number of items used from each part of the inventory to satisfy the current demand (i.e., the demand for period t, which is known) at period t. Therefore, $\chi^t = (y_1^t, ..., y_N^t, x_{21}^t, ..., x_{2,M-1}^t, ..., x_{N,1}, ..., x_{N,M-1}^t, a_{1,1}^t, ..., a_{1,M}^t, ..., a_{N,1}^t, ..., a_{N,M}^t)$. The action space is limited by $0 \leq a_{ij}^t \leq b_{i,j}^t$, $y_i^t > 0$ and $0 \leq x_{ij}^t \leq I_{i,j}^t$. $b_{i,j}^t$ is the total inventory with age j at hospital i at the beginning of the period t before fulfilling the demand and making the transshipment. Later, to solve the dynamic programming formulation, $a_{i,j}^t$'s was removed from the actions, as the state of the system in the next period ($S^{t+1} = (I_1^{t+1}, ..., I_N^{t+1})$) only depends on the state of the system in the current period ($S^t = (I_1^t, ..., I_N^t)$). This is further clarified when the model is formulated in (3.13).

The model defines the value function ($V(S)$) for state S, which specifies the minimum discounted cost over the infinite horizon and should satisfy the following optimality equations, which are known as Bellman equations:

$$V(S^t) = \min \; C^t(S^t, \chi^t) + \lambda \sum_{S^{t+1}} p(S^{t+1}|S^t, \chi^t)V(S^{t+1}) \qquad \forall S^t. \tag{3.6}$$

Where is the discount factor.

3.4.1 Approximate dynamic programming

Traditional methods (e.g., the value iteration or the policy iteration) for solving (3.6) are not applicable here because of the dimension of the state and action spaces. According to Puterman (1994), solving the optimality in Equation (3.6) is equivalent to solving the following LP for any strictly positive α:

$$\begin{aligned} & \max \sum_{S} \alpha(S)V(S) \\ & \text{subject to} \\ & V(S) \leq C(S, \chi) + \lambda \sum_{\acute{S}} p(\acute{S}|S, \chi)V(\acute{S}) \qquad \forall S, \chi. \end{aligned} \tag{3.7}$$

Without loss of generality, $\alpha(S)$ is considered a probability distribution over the initial state of the system (Puterman (1994, 1, p. 223)). The above model has a variable for every state and a constraint for every state-action pair, so converting to an LP does not avoid the curse of dimensionality. One possible approach for solving the model is to approximate the value function, with a linear combination of basis functions. The following approximation to the value function is defined as follows:

$$V(S) = \omega_0 + \sum_{i \in \mathcal{N}} \sum_{j=1}^{M} I_{i,j}\omega_{ij}. \tag{3.8}$$

ω_{ij} can be interpreted as the marginal cost for each additional unit of age j.

Then, (3.8) can be replaced into (3.7) to find the best value of (ω_0, ω_{ij}), such that (3.8) would be an acceptable approximation of value function.

$$
\begin{aligned}
&\max \quad \omega_0 + \sum_{i \in \mathcal{N}} \sum_{j=1}^{M} E_\alpha[I_{i,j}]\omega_{ij} \\
&\text{subject to} \\
&(1-\lambda)\omega_0 + \sum_{i \in \mathcal{N}} \sum_{j=1}^{M} W_{ij}(S,\chi)\omega_{ij} \leq C(S,\chi) \quad \forall S, \chi \\
&\text{where,} \\
&E_\alpha[I_{i,j}] = \sum_{S} \alpha(S) I_{i,j}(S) \qquad \forall i \in \mathcal{N},\ j \in \{1, 2, \ldots, M\} \\
&W_{ij}(S,\chi) = I_{i,j}(S) - \lambda \sum_{\acute{d}_i \in \mathcal{D}} P(\acute{d}_i) I'_{i,j} \qquad \forall i \in \mathcal{N}, j \in \{1, 2, \ldots, M\}.
\end{aligned}
\tag{3.9}
$$

where D is the set of all possible incoming demand streams.

Although the above approximation reduces the number of decision variables (as it has $NM + 1$ variables), the model in (3.9) still has a large number of constraints because there is one constraint for each possible state-action pair. Based on the fundamental theorem of linear programming to obtain the optimal solution, it is sufficient to discover at most, $NM + 1$ of these constraints, which can be binding on optimality. After this, an initial set of $NM + 1$ constraints can be used to find the most violated constraint and add to the initial set. Adding new constraints can be halted when the optimality gap is smaller than the desired precision. This is equivalent to solving the dual of the above problem by utilizing delayed column generation (Adelman (2007)). The dual of problem (3.9) is as follows:

$$
\begin{aligned}
&\min \qquad \sum_{S,\chi} C(S,\chi)\rho(S,\chi) \\
&\text{subject to} \\
&(1-\lambda) \sum_{S,\chi} \rho(S,\chi) = 1 \\
&\sum_{S,\chi} W_{ij}(S,\chi)\rho(S,\chi) = E_\alpha[I_{i,j}] \qquad \forall i \in \mathcal{N}, j \in \{1, 2, \ldots, M\} \\
&\rho(S,\chi) \geq 0 \qquad \forall S, \chi
\end{aligned}
\tag{3.10}
$$

The problem here is that while (3.10) has a variable for each state-action pair, at most, $NM+1$ of those variables need to be non-zero at optimality. Consequently, it is possible to use the delayed column generation technique and to add new variables as required.

If there is an initial set of columns for the problem in (3.10), it is possible to solve the relaxed dual problem and to determine its optimal solution as well as the corresponding optimal solution of the relaxed primal problem (see the model in (3.9)) (i.e., $(\omega_0^*, \omega_{ij}^*) \quad \forall i \in \mathcal{N}, j = 1, \ldots, M$). This solution may not satisfy all of the constraints of the primal problem because not all of the constraints of the original problem have been considered. The most violated constraint is found by solving a pricing sub-problem that finds a feasible state-action pair (S, χ) that maximizes

$$\tilde{V}(S) - C(S,\chi) - \lambda \sum_{\acute{S}} p(\acute{S}|S,\chi)\tilde{V}(\acute{S}) \qquad \forall S. \tag{3.11}$$

If the maximum in (3.11) for the optimal state-action pair (S^*, χ^*) is greater than 0, this implies that the corresponding constraint of this state-action pair is the most violated constraint. Alternatively, if the maximum is not positive, then the current optimal solution $((\omega_0^*, \omega_{ij}^*), \quad \forall i \in \mathcal{N}, j = 1, \ldots, M)$ satisfies all of the constraints of the original primal problem.

3.4.2 Procedure to determine the best value of (ω_0, ω_{ij})

To approximate the coefficients (ω_0, ω_{ij}), the dual of the (3.9) should be solved by using delayed column generation. To find the initial set of columns that produces a feasible basic solution to the problem in (3.10), the Phase I method of linear programming can be used.

To start the Phase I method, an artificial variable is added to each constraint of the problem (3.10) so as to create an identity matrix of size $(NM+1)$. Consequently, the sum of these artificial variables should be minimized by generating and adding columns from the original dual problem (3.10). This approach of adding columns to the Phase I problem is similar to the approach for adding new variables to the model presented in (3.10), which is discussed next.

When the Phase I objective function becomes zero, the columns corresponding to these

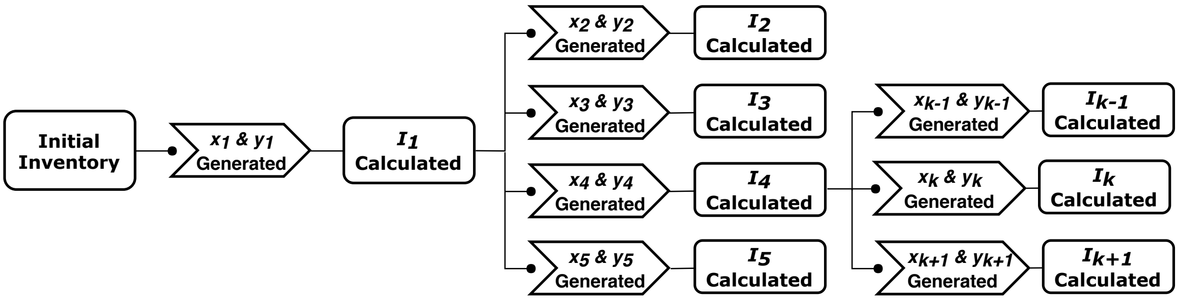


FIGURE 3.2: Sampling state procedure.

artificial variables (the identity matrix) can be eliminated and the remaining columns provide a feasible solution to the problem (3.10).

Considering this initial set of columns for model (3.10), the optimal solution is found, as well as the corresponding optimal dual solution $(\omega_0^*, \omega_{ij}^*)$.

In (3.10), $E_\alpha[I_{i,j}]$ is estimated by sampling from the states. Generating each sample state, the initial inventory level is considered first, to generate different random action plans and demands. The policy for generating transshipment decisions is to transship the oldest item in the system; this new model for generating y can pick a random number for order quantity, from zero to 2 times of generated demand for the current state. In addition, to satisfy the demand for the current period, a policy that fulfills the demand from the oldest available items can be applied; hence, $a_{i,j}$ are decided according to a FIFO policy.

For each pair of (x, y, a), new state $(I_i = (I_{i,1}, ..., I_{i,M}))$ is found. The above method for new state is continued until there is enough (I_i). Refer to the procedure for calculating $E_\alpha[I_{i,j}]$ shown in Figure 3.2.

To generate a new variable for (3.10), the following sub-problem is solved. Which finds a feasible state-action pair (S, χ):

$$\begin{aligned}
&\max \quad (1-\lambda)\omega_0^* + \sum_{i\in\mathcal{N}}\sum_{j=1}^{M} W_{ij}(S,\chi)\omega_{ij}^* - C(S,\chi) + g_i(\mathbb{E}(F_i^{'}) - \sum_{i=1}^{N}\sum_{d_i^{'}\in\ \mathcal{D}_i} p(d_i^{'})(F_i^{'}(d_i^{'}))\\
&\text{where,}\\
&W_{ij}(S,\chi) = I_{i,j} - \lambda \sum_{d_i^{'}\in\mathcal{D}_i} P(d_i^{'})(I_{i,j-1} - a_{i,j}^{'}(d_i^{'}) - x_{i,j-1}), \qquad \forall i \in \mathcal{N}\setminus\{1\}, j = \{2, ..., M\};\\
&W_{i1}(S,\chi) = I_{i,1} - \lambda \sum_{d_i^{'}\in\mathcal{D}_i} P(d_i^{'})(y_i - a_{i,1}^{'}(d_i^{'})), \qquad \forall i \in \mathcal{N};\\
&W_{1j}(S,\chi) = I_{1,j} - \lambda \sum_{d_1^{'}\in\mathcal{D}_i} P(d_1^{'})(I_{1,j-1} + \sum_{i\in\mathcal{N}\setminus\{1\}} x_{i,j-1} - a_{1,j}^{'}(d_1^{'})), \qquad j = \{2, ..., M\}\\
&a_{i,j}^{'}(d_i^{'}) \le I_{i,j-1} - x_{i,j-1}, \qquad \forall i \in \mathcal{N}\setminus\{1\}, j = \{2, ..., M\}, \forall d_i^{'} \in\ \mathcal{D}_i;\\
&\sum_{j=1}^{M} a_{i,j}^{'}(d_i^{'}) + F_i^{'}(d_i^{'}) = d_i^{'}, \qquad \forall i \in \mathcal{N}, j = \{1, ..., M\}, \forall d_i^{'} \in\ \mathcal{D}_i;\\
&a_{1,j}^{'}(d_i^{'}) \le I_{1,j} + \sum_{i=2}^{N} x_{i,j-1}, \qquad j = \{2, ..., M\}, \forall d_i^{'} \in\ \mathcal{D}_i;\\
&a_{i,1}^{'}(d_i^{'}) \le y_i, \qquad \forall i \in \mathcal{N}, \forall d_i^{'} \in\ \mathcal{D}_i;\\
&x_{i,j} \le I_{i,j}, \qquad \forall i \in \mathcal{N}\setminus\{1\}, j = 1, .., M;\\
&\sum_{j=1}^{M} I_{i,j} - \sum_{j=1}^{M-1} x_{ij} + y_i \ge E(D_i), \quad \forall i \in \mathcal{N}\setminus\{1\};\\
&\sum_{j=1}^{M} I_{1,j} - \sum_{j=1}^{M-1} x_{1j} + y_1 \ge E(D_1);\\
&y_i \in Z^+, \quad i \in \mathcal{N};\\
&x_{i,j}, a_{i,j}^{'}(d_i^{'}), I_{i,j} \in Z^+, \quad i \in \mathcal{N}, j = \{1, 2, ..., M\};
\end{aligned} \tag{3.12}$$

$a_{i,j}^{'}(d_i^{'})$ are the items used to meet the next period demand (which is unknown); therefore, they depend on $d_i^{'}$ (notation $(d_i^{'})$ indicates that they are function of $d_i^{'}$). In the model (3.12), the decision variables are the state $(I_1^t, \cdots, I_N^t)$ and the action $\chi = (y_1, y_2, ..., y_N, x_{2,1}, ..., x_{2,M-1}, x_{3,1}, ..., x_{3,M-1}, ..., x_{N,M-1})$. Note that $a_{i,j}^t$ are excluded from the action space, as the aim here is to estimate the value function.$a_{i,j}^t$ are used to compute the current state and they can be added later in decision-making procedure after estimating the value functions. $\mathbb{E}(F_i^{'})$ is a fixed value estimated as part of $C(S, \chi)$. However, it needs to be linked $F_i^{'}$ to $a_{i,j}^{'}$; hence $\mathbb{E}(g_i F_i^{'})$ is removed from $C(S, \chi)$ and added as $g_i \sum_{i=1}^{N} \sum_{d_i^{'} \in \mathcal{D}_i} p(d_i^{'})(d_i^{'} - \sum_{j=1}^{M} a_{i,j}^{'})$. In addition, the following constraints are added to the model (3.12) to limit the feasibility region $I_{i,j} - \sum_{j=1}^{M-1} x_{ij} + y_i \geq E(D_i)$.

If the maximum of (3.12) is greater than zero, the constraint corresponding to this state- action pair is the most violated constraint. Therefore, the new variable with a coefficient corresponding to this state-action pair is added to the model (3.10) and it is resolved to obtain a new solution. Then, this new solution is used to find another violated constraint. The process of adding more constraints is halted when no more constraints are violated.

Conversely, if the maximum of (3.12) is not positive, the current solution $(\omega_0^*, \omega_{ij}^*)$ satisfies all the constraints and is the optimal solution. Figure 3.3 shows the flowchart of the procedure used to determine the best value of (ω_0, ω_{ij}).

3.4.3 Deriving transshipment and ordering policy

In the previous section, the approach for determining the best approximate value of the value function was presented. At the beginning of each day (t), the inventory at the beginning of the period (i.e., $b_{i,j}^t$'s) is known, as well as the demand for the current period (d_i^t). Decisions on y_i^t, $x_{i,j}^t$ and $a_{i,j}^t$ can then be made, thus solving the following

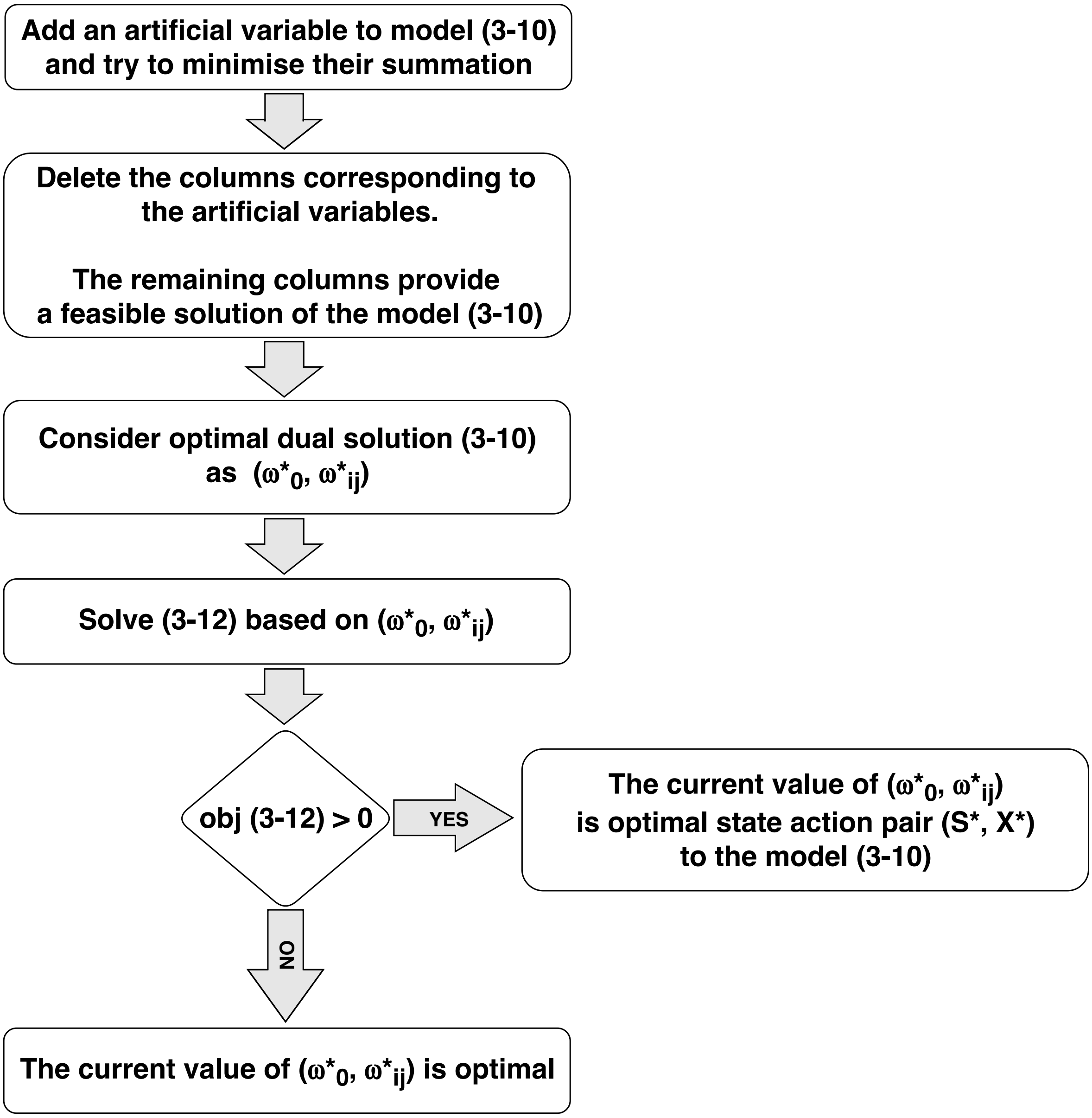


FIGURE 3.3: Procedure to determine the best value of (ω_0, ω_{ij}).

problem at the beginning of each period t:

$$
\begin{aligned}
&\min \quad \sum_{i\in\mathcal{N}}\Bigg(c_i y_i^t + o_i I_{i,M}^t + h_i(y_i^t + \sum_{j=1}^{M-1} I_{i,j}^t) + g_i F_i^t + g_i \sum_{d_i^{t+1}\in\ \mathcal{D}_i} p(d_i^{t+1})(F_i^{t+1}(d_i^{t+1}))\\
&+\sum_{i\in\mathcal{N}\setminus\{1\}}\sum_{j=1}^{M-1} q_i^t x_{ij}^t\Bigg)+\\
&\lambda\Bigg[\sum_{i\in\mathcal{N}\setminus\{1\}}\sum_{d_i^{t+1}\in\mathcal{D}_i} p(d_i^{t+1})(\omega_0+\sum_{j=2}^{M}(I_{i,j-1}^t - a_{i,j}^{t+1}(d_i^{t+1}) - x_{i,j-1}^t)\omega_{ij})\\
&+\sum_{i\in\mathcal{N}}\sum_{d_i^{t+1}\in\mathcal{D}_i} p(d_i^{t+1})((y_i^t - a_{i,1}^{t+1}(d_i^{t+1}))\omega_{i1})\\
&+\sum_{d_1^{t+1}\in\mathcal{D}_1} p(d_1^{t+1})(\omega_0+\sum_{j=2}^{M}(I_{1,j-1}^t + \sum_{i\in\mathcal{N}\setminus\{1\}} x_{i,j-1}^t - a_{1,j}^{t+1}(d_1^{t+1}))\omega_{1j})\Bigg]\\
&\text{Subject to:}\\
&x_{ij}^t \le I_{i,j}^t, \qquad i\in\mathcal{N}\setminus\{1\}, \quad j\in\{1,2,\dots,M-1\};\\
&a_{ij}^t \le b_{i,j}^t \qquad i\in\mathcal{N}, \quad j\in\{1,2,\dots,M\};\\
&d_i^t = F_i^t + \sum_{j=1}^{M} a_{ij}^t \qquad i\in\mathcal{N};\\
&I_{i,j}^t = b_{i,j}^t - a_{ij}^t \qquad i\in\mathcal{N};\\
&d_i^{t+1} = \sum_{j=1}^{M} a_{i,j}^{t+1}(d_i^{t+1}) + F_i^{t+1}(d_i^{t+1}), \qquad i\in\mathcal{N}, \quad j\in\{1,2,\dots,M\};\\
&a_{i,j}^{t+1}(d_i^{t+1}) \le I_{i,j-1}^t - x_{i,j-1}^t,\ i\in\mathcal{N}\setminus\{1\}, j=\{2,...,M\}, \forall d_i^{t+1}\in\ \mathcal{D}_i;\\
&a_{1,j}^{t+1}(d_i^{t+1}) \le I_{1,j}^t + \sum_{i=2}^{N} x_{i,j-1}^t, j=\{2,...,M\}, \forall d_i^{t+1}\in\ \mathcal{D}_i;\\
&a_{i,1}^{t+1}(d_i^{t+1}) \le y_i^t,\ i\in\mathcal{N}, \forall d_i^{t+1}\in\ \mathcal{D}_i;\\
&y_i^t, F_i^t \in Z^+, \quad i\in\mathcal{N};\\
&x_{i,j}^t, a_{i,j}^{t+1}(d_i^{t+1}), a_{i,j}^t \in Z^+, \quad i\in\mathcal{N}, j=\{1,2,...,M\};
\end{aligned}
\tag{3.13}
$$

Note that $g_i\sum_{i\in\mathcal{N}} F_i^t$ could be part of the immediate cost of the action that was initially from the definition of the immediate cost, to be able to calculate the value functions. To determine the transshipment and order quantities, first the daily demand is generated randomly and then the optimal order and transshipment policy is based on the current inventory and demand. Next, the inventory is updated, considering the quantity of transshipment and orders, and the same procedure is repeated for the next period.

3.5 Numerical results

A network of three hospitals (one main hospital and two small hospitals) was considered. The performance metrics reported for each of these were based on simulating 18,500 days of this network. The system could be run by applying the proposed model for 18,500 days and solving the model 18,500 times for each case. The number of simulation runs was set according to the Dvoretzky-Kiefer-Wolfowitz inequality (Abbasi et al. (2018)), which gives an estimate of the total number of simulations required to attain an empirical cumulative distribution of cost components with an error less than 0.01%. Each simulation run takes almost two seconds on a typical personal computer. The algorithm was implemented in Python 2.7.10 and the ADP models were solved using IBM ILOG CPLEX 12.6.2.

As the actual demand forecasts for red blood cells could not be made available, the daily demand for blood was generated randomly. Note that daily demand varied during the week, with significantly less (and often zero) demand on weekends. Thus, it was assumed that the uncertain demand followed a zero-inflated negative binomial distribution. The reason for assuming this distribution is that the negative binomial distribution is a flexible discrete distribution and can take the index of dispersion to over one. Previous research has noted that the distribution of demand for blood components has an index of dispersion over one, and therefore, using Poisson distribution (known to have an index of dispersion equal to one) underestimates the demand variability (Goh, Greenberg, and Matsuo (1993) and Abbasi and Hosseinifard (2014)). In addition, as there may be no demand on some days, especially on weekends (as only emergency operations are scheduled then), zero-inflated negative binomial distribution was used, to inflate the probability of observing zero demand. The following parameters were used for zero-inflated negative binomial distribution in these experiments $n = [25, 25, 25], p = [0.60, 0.65.0.40], c = [0.6, 0.6, 0.3]$.

The total shelf life of the blood units was set at 21 days, in agreement with the shelf life of red blood cells at some blood services (Flegel, Natanson, and Klein (2014)). Note that in these experiments, the average age of red blood cells issued to a hospital was assumed at 10 days (i.e., on average the units were held for 10 days in the collection centers, processing centers, and blood banks before being issued to a hospital). Therefore, the remaining shelf life of the units received by the hospitals was set at 11 days.

All cost components were considered equal at all hospitals and the value of the costs of holding, expiry, emergency order, order and transshipment were set at 1, 16, 13, 3, and 1.5 per unit, respectively.

The performance of the proposed model was explored by comparing it to the current policy practiced in some Australian hospitals, which apply a daily review inventory policy. This means the stock status is monitored every day and, if the inventory level falls below a specific inventory level S, an order is placed to raise the inventory up to S. The units are issued to fulfill the demand in a FIFO policy. In terms of the new transshipment policy being developed in this study, the small-sized hospitals were allowed to transship their units of red blood cells that had an age greater than 15 days to large-sized hospitals within their network. This study assumed that Hospitals 1 and 2 were both small-sized hospitals that could transship their red blood cells with less than 14 days residual shelf life to Hospital 3 .

Henceforth in this section , the name *ADP Model* is used for the proposed new model, *Current Policy* is applied to the model that simulates the current policy practiced at some Australian hospitals, and *No Transshipment* is used when the hospitals apply the same inventory policy (base stock) as the current policy but they do not apply transshipment.

In the first set of experiments, the three aforementioned models were compared. The results acquired in terms of average daily outdate (Avg.Out) and average daily shortage (Avg.Short) for each hospital, are presented in Table 3.1, while Table 3.2 shows the average daily component costs.

As shown in Tables 3.1 and 3.2, the three policies had comparable performance in the smaller hospitals (i.e., Hospitals 1 and 2), in terms of outdate. Table 3.1 shows that the transshipment had a positive effect when the results for the ADP Model and Current Policy were compared to No Transshipment, in terms of wastage of blood units.

Hospitals	ADP Model		Current Policy		No Transshipment	
	Avg.Short	**Avg.Out**	**Avg.Short**	**Avg.Out**	**Avg.Short**	**Avg.Out**
Hospital 1	0.997	0.000	0.126	0.000	0.016	0.450
Hospital 2	0.251	0.000	0.076	0.000	0.010	0.475
Hospital 3	2.073	0.000	0.008	0.084	0.017	0.013
Total	3.322	0.000	0.210	0.084	0.043	0.938

Table 3.1: Average of shortage and outdate for each hospital (shortage cost per unit=16, outdate cost per unit=15).

	ADP Model	Current Policy	No Transshipment
Total cost parcels			
Holding cost	22.990	126.834	122.788
Order cost	104.643	114.219	117.320
Shortage cost	53.145	3.364	0.685
Outdate cost	0.002	1.256	14.064
Transshipment cost	15.274	6.150	—
Total cost statistics			
Average	196.055	251.823	254.858
Std. dev.	111.500	62.788	68.730
Median	154.500	250.000	250.000
Skewness	1.859	1.670	1.355
P5	98.500	167.500	158.000
P1	94.000	142.000	134.000

Table 3.2: The average daily component costs (shortage cost per unit=16, outdate cost per unit=15). P1 and P5 denote the first percentile and the fifth percentile respectively.

Table 3.2 presents the average daily component costs for all the models. It shows that all the components cost for the ADP Model were lower than those for Current Policy, except for the shortage and transshipment costs. Further, the average total cost for the ADP Model was nearly 31% less than that obtained with Current Policy and No Transshipment. Moreover, it was clear that the policy devised by the ADP Model was much more efficient with regard to expected costs. The statistical information of the distribution of cost enabled a presumption that the ADP Model was less subject to scenarios of high costs.

To verify these results, the outdate cost was changed to 14, the shortage cost to 18, and the other cost parameters were set the same as in the previous example. According to Table 3.3, changing the outdate and shortage costs did not affect the daily average shortage and outdate costs results for Current Policy and No Transshipment. Further, the results in Table 3.3 and Table 3.4 showed that the ADP Model had consistently superior performance.

Hospitals	**ADP Model**		**Current Policy**		**No Transshipment**	
	Avg.Short	**Avg.Out**	**Avg.Short**	**Avg.Out**	**Avg.Short**	**Avg.Out**
Hospital 1	0.997	0.000	0.126	0.000	0.016	0.450
Hospital 2	0.251	0.000	0.076	0.000	0.010	0.475
Hospital 3	2.024	0.000	0.008	0.084	0.017	0.013
Total	3.272	0.000	0.210	0.084	0.043	0.938

Table 3.3: Average of shortage and outdate for each hospital (shortage cost per unit=18, outdate cost per unit=14).

	ADP Model	**Current Policy**	**No Transshipment**
Total cost parcels			
Holding cost	23.133	126.834	122.788
Order cost	104.791	114.219	117.320
Shortage cost	58.905	3.785	0.771
Outdate cost	0.002	1.172	12.189
Transshipment cost	15.319	6.150	—
Total cost statistics			
Average	202.150	252.160	253.068
Std. dev.	115.295	58.525	68.730
Median	154.500	250.000	250.000
Skewness	1.932	1.674	1.055
P5	99.500	167.000	158.000
P1	94.500	142.000	134.000

Table 3.4: The average daily component costs (shortage cost per unit=18, outdate cost per unit=14). P1 and P5 denote the first percentile and the fifth percentile respectively.

A sensitivity analysis was performed, to analyze the effect of transshipment cost on the system. The transshipment cost was changed from 0 to 2 and the other cost parameters were held the same as for the first example. The results of this experiment are presented in Table 3.5. The results demonstrated that increasing the transshipment cost did not affect the average daily shortage, average daily outdate and service level of the Current Policy scenario because it did not affect that transshipment policy. Additionally, as the transshipment cost increased, the average daily shortage in the ADP Model marginally increased.

	ADP Model			Current Policy		
Transshipment cost	**Avg.Short**	**Avg.Out**	**Avg.cost**	**Avg.Short**	**Avg.Out**	**Avg.cost**
0	3.223	0.000	179.737	0.210	0.084	245.673
0.5	3.249	0.000	185.183	0.210	0.084	247.723
1	3.274	0.000	190.487	0.210	0.084	249.773
1.5	3.322	0.000	196.055	0.210	0.084	251.823
2	3.358	0.000	201.354	0.210	0.084	253.873

Table 3.5: Average of shortage, outdate, and cost for different transshipment cost (shortage cost per unit=16, outdate cost per unit=15).

3.6 Summary

In this chapter, a model has been proposed to decide on proactive transshipment to balance the existing trade-off between wastage and shortages, as well as to reduce the total cost. A blood inventory system for a network of hospitals with uncertain demand was considered. A dynamic programming model was applied to obtain the optimal quantities of order and transshipment to minimize the total expected cost. This dynamic programming formulation suffered from the curse of dimensionality. To tackle this difficulty, ADP techniques were applied. A set of basis functions were defined to approximate the value function of the dynamic programming. A column generation algorithm was used to solve the linear programming form of the dynamic programming.

The performance of the proposed model was evaluated through numerical experiments comparing it with the performance of the current transshipment policy applied in some hospitals. The numerical results have highlighted that significant cost benefits could be obtained through the balance between shortage and wastage that is achieved by using the ADP Model, as well as illustrating the advantages of proactive transshipment in the blood supply chain.

Chapter 4

Proactive Transshipment in the Blood Supply Chain: a Stochastic Programing Approach

4.1 Introduction

This chapter shows the way transshipment can be proactively considered in an inventory control system as a powerful mechanism to rebalance the blood inventory in a network of hospitals and ultimately, to reduce the costs associated with product shortage, outdate, holding, ordering, and transshipment.

In the context under study, it is considered that the hospitals optimally decide the amount to be ordered and transshipped every-day, with the support of the proposed model every-day. The proactive transshipment decisions are made at the same time that the hospitals place their orders to replenish their inventories. The perspective of a centralized planner that manages several hospitals simultaneously is assumed, in a system that consists of a network of hospitals facing uncertain demand, with general probability distribution. Therefore, at the beginning of each period and before demand is known (i.e., observed), the planner can decide to replenish the inventory at the hospitals either by placing orders to the blood bank or via transshipment from the other hospitals in the network.

To model the aforementioned context, a mathematical model using two-stage stochastic programming (2SSP) is developed in a rolling-horizon framework, to devise optimal ordering and transshipment policies for a blood inventory system. Stochastic programming is a framework that can be used to model optimization problems with uncertain input parameters and thus, is well suited for characterizing real-world problems and their inherent uncertainties. Dantzig (1955) introduced 2SSP in the 1950s for the first time, to handle uncertainty in mathematical programming. Since then, it has been studied extensively, both in theory and with regard to computational aspects. Some examples of 2SSP applications in inventory management are discussed in the following literature review section.

In the standard form of a 2SSP model, decision variables are divided into two groups, namely first- and second-stage decisions. First-stage decisions must be made before the actual realization of the random (uncertain) parameters. Second-stage decisions are made when the uncertain parameters have been unveiled. The goal in this framework is to find values for the first-stage decisions that are feasible for all (or almost all, in the case where probabilistic constraints are used) scenarios and to optimize the objective function with respect to current and expected future costs.

In the 2SSP framework, uncertainty is represented by a finite set of scenarios that approximate the original stochastic phenomenon. Since most of the methods of scenario generation involve randomness, the result must be stable with respect to the scenario sample used, meaning that if several samples of scenario sets are generated and the optimization problem with these sets are solved, then similar optimal values for the objective function and decision variables should be observed. A QMC method was chosen for this research, to generate scenarios and reach stability without having to consider a prohibitive number of discrete scenarios. A stability analysis was performed to confirm the suitability of the scenario set generated. To examine the dynamics of the proposed model, these scenarios were generated using realistic data for the daily demand of blood, based on the average and standard deviation for one type of blood.

Multistage stochastic programming is a generalization of the 2SSP, which is naturally suited to representing the dynamics of the problem at hand. To avoid problems related to having multiple decision stages and ultimately, to make the problem computationally tractable (i.e., solvable in a reasonable time), multistage stochastic programming models are often reformulated and approximated by 2SSP models. In this research, this was achieved by combining two central ideas. First, a simplified approximation of the future (i.e., second-stage) decisions was made. Second, a rolling-horizon method was used. Thus, every decision stage (i.e., days in the planning horizon) could be solved as a 2SSP model where its inputs were decisions from the previous stage (initial inventory and age profile), and future decisions could be represented by this simplified future approximation. Section 4.3 describes the simplifications that were made and the way they were incorporated into this rolling-horizon framework.

The remainder of this chapter is structured as follows. Section 4.2 provides a review of the related literature on blood inventory management. Section 4.3 presents a detailed description of the proposed model. Section 4.4 presents the mathematical formulation of the 2SSP model. Section 4.5 contains the numerical study and Section 4.6 offers conclusions.

4.2 Literature review

As blood is a precious perishable commodity, many researchers have focused on the management of blood. Nahmias (1982) and Prastacos (1984) presented a review of early research in blood inventory management and Beliën and Forcé (2012) published a review paper of blood inventories and supply chain management. More recently, Osorio, Brailsford, and Smith (2015) provided a comprehensive literature review of quantitative models for the management of the blood supply chain.

A variety of methodologies, such as queuing theory and Markov chains have been used, alone or in combination, to analyze blood products. Pegels and Jelmert (1970) and Abbasi and Hosseinifard (2014) used Markov chains to investigate the effects of modified FIFO and LIFO issuing policies on the average on-hand inventory and the average age of issued blood. Brodheim, Derman, and Prastacos (1975) proposed a fixed-order-quantity model for perishable products and formulated the problem as a Markov chain to compute the average age of blood in inventory and the probability of blood shortage. Cumming et al. (1976) proposed a Markovian population model that aimed to keep the balance between supply and demand of blood. Kopach, Balcıoğlu, and Carter (2008) applied a queuing framework to model a red blood cell inventory system with two demand levels: urgent and non-urgent demand. Hosseinifard and Abbasi (2018) used a queuing theory framework with a Poisson demand distribution to evaluate the effects for the blood supply chain of inventory pooling.

Simulation has been frequently used as a method to optimize the blood supply chain because of the complexity of blood inventory problems, despite the fact that the optimality of the solution obtained could not be guaranteed. Rytilä and Spens (2006) used discrete event simulation to improve efficiency in the blood supply chain. Duan and Liao (2013) developed a new replenishment policy based on old inventory ratio for highly perishable items. They applied a simulation-optimization approach to optimize replenishment policies. Katsaliaki and Brailsford (2007) used discrete event simulation to minimize costs, shortage, and wastage in the blood supply chain by determining optimal ordering policies. Mustafee et al. (2009) improved their model by proposing a distributed simulation approach to reduce simulation run time. Kamp et al. (2010) used simulation methods to study the availability of blood products in pandemic situations in Germany. Abbasi, Vakili, and Chesneau (2017) and Blake et al. (2013b) used simulation modeling to evaluate the effect of reducing the shelf life of red blood cells in Australian and Canadian blood supply chains, respectively.

Statistical analysis methods, such as linear regression, survival analysis, and logistic regression, have been used to support decision making in the blood supply chain. Melnyk et al. (1995) used survival analysis to classify blood donors and increase donor satisfaction by improving the layout of collection stations. Bosnes, Aldrin, and Heier (2005) used a logistic regression model to predict blood donor arrivals. Godin et al. (2007) applied a logistic regression model to obtain the main factors with regard to repeated blood donation. Heddle et al. (2009) applied logistic regression techniques to determine factors that affected outdating of red blood cells. Perera et al. (2009) analyzed blood stock, using the t-test and determined factors affecting stock level and wastage.

The complexity of the problems in blood inventory management means that optimization methods such as stochastic dynamic programming, integer programming, and linear programming have been used less frequently. Haijema et al. (2009) proposed a stochastic dynamic programming and simulation approach to design optimal order-up-to-level inventory policies for platelet production. Kendall and Lee (1980) proposed a goal programming model to allocate blood units to hospitals and minimize wastage. They evaluated solutions based on stock availability, the age of blood, the outdate rate, and the availability of fresh blood. Pitocco and Sexton (2005) used a data envelopment analysis model to evaluate the efficiency of 70 blood centers. Hemmelmayr et al. (2009) proposed an integer programming model to determine optimal delivery days by minimizing wastage and delivery costs. They considered the known daily demand for each hospital and investigated whether switching from the current vendee-managed inventory system to a vender-managed inventory system could be beneficial. Şahin, Süral, and Meral (2007) developed an integer programming model to address the location-allocation decision problems in the regionalization of blood services. They considered the total population of cities as demand for blood and validated their models by using real data for Turkish Red Crescent blood services. Gunpinar and Centeno (2015) used an integer programming model to minimize the total costs (shortage, outdating, holding, and purchasing) for a single-level inventory system with uncertain demand.

Two-stage stochastic programming has being considered a suitable framework for inventory management problems other than blood supply chain management. Fattahi et al. (2015) used two-stage stochastic programming to study a multi-period replenishment problem under centralized and decentralized supply chain systems. They assumed a safety-stock-based policy with uncertain demand for a supply chain consisting of one retailer and one manufacturer. Cunha, Raupp, and Oliveira (2017) developed a two-stage stochastic programming model to determine the optimal strategies of a replenishment control system considering uncertain demand and periodic review. Dillon, Oliveira, and Abbasi (2017) proposed a two-stage stochastic programming model to manage red blood cells inventory. They considered periodic-review policies with a fixed ordering point and minimized the total cost, as well as shortage and wastage, considering uncertain demand.

The aforementioned studies focused on applying lateral transshipment to improve the performance of a non-perishable inventory. However, there is a gap in the literature with regard to the effects of transshipment on perishable inventory, especially blood inventory. Dehghani and Abbasi (2018) developed a model for the lateral transshipment of blood products. Their model was limited to a Poisson demand distribution and only worked for the transshipment of perishable items between two locations. This current study appears to be the first to consider lateral transshipment of perishable items (e.g., blood products) in a network of inventory locations (e.g., multiple hospitals), without being restricted to a specific demand distribution.

4.3 Problem setting and preliminaries

In this part of the research, a network with N hospitals is considered, denoted by index $i \in \mathcal{N} := \{1, 2, \ldots, N\}$. The planning horizon is divided into T periods of uniform length (generally representing days), denoted by $t \in \mathcal{T} := \{1, 2, \ldots, T\}$. The hospitals considered as references for this study employ a periodic inventory review policy, denoted by (R, S), meaning that at each review point (e.g., every R-th morning), the hospital reviews the inventory status and places an order to raise the inventory level to the target S. The (R, S) policy with $R = 1$ is currently used (for small hospitals, $R = 2$ is used as well) in some hospitals in Australia because of its reasonable efficiency and simplicity (Abbasi, Vakili, and Chesneau (2017)). However, in this study, the proposed model allows for the use of a dynamic periodic inventory policy and thus, does not enforce the (R, S) policy.

Further, it is assumed that all hospitals place their orders with the CBB for new batches of blood units at the start of each period $t \in \mathcal{T} := \{1, 2, \ldots, T\}$. The order quantity of hospital $i \in \mathcal{N}$ at the beginning of period t is denoted by y_i^t. Ordering from the CBB to hospital $i \in \mathcal{N}$ costs R_i per blood unit. While this model does not assume fixed costs, the adaptation to include such costs would be straightforward from a modeling perspective and would require the inclusion of binary variables to capture the ordering event, as in Dillon, Oliveira, and Abbasi (2017). Each batch of blood is assumed to have a shelf life of M periods. Therefore, if some of the blood units at hospital i are not used within M periods, they must be discarded, incurring an expiry cost E_i. In addition, it is assumed that the orders can only be used to fulfill future demand, starting from the next day (i.e., with a lead time of one period), as under this study, it takes up to one day for the blood units to arrive in the setting. Hence, orders placed in period t can only be used to fulfill demands from period $t + 1$ onwards.

Hospitals can transship blood from their inventory to other hospitals in the network. This model denotes the transshipment amount in period 1 from hospital i to hospital j, $i, j \in \{\mathcal{N} : i \neq j\}$, by $x_{ij}^1 := (x_{ij1}^1, x_{ij2}^1, \ldots, x_{ijM}^1)$, where x_{ijm}^1, $m \in \{1, 2, \ldots, M\}$ denotes the amount of transshipped blood units that have remaining shelf life m at the beginning of period 1. Transshipment from hospital i to hospital j costs C_{ij} per blood unit. At the end of period t, each hospital $i \in \mathcal{N}$ observes a random demand D_i^t. If the hospital does not have enough inventory, it can place an emergency order to fulfill the excess demand. Emergency-order, costs G_i per unit to hospital i. If hospital $i \in \mathcal{N}$ has an excess of inventory after fulfilling all its demand, a holding cost of H_i per unit is incurred for unexpired blood units at the end of period t.

$B_i^1 := (B_{i1}^1, B_{i2}^1, \ldots, B_{iM}^1)$ denotes the inventory at hospital $i \in \mathcal{N}$ at the beginning of the first period. B_{im}^1, representing the quantity of blood that has a remaining shelf life of $m \in \{1, 2, \ldots, M\}$ periods at the beginning of period 1. It assumes that the inventory at the beginning of the current period (i.e., $t = 1$) is known. It represents the quantity of shortage in period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$ with f_i^t and denotes the total inventory at hospital $i \in \mathcal{N}$ at the end of period $t \in \mathcal{T}$ with v_i^t. In addition, it denotes the amount of outdated blood in period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$ with o_i^t.

Figure 4.1 illustrates the dynamics of the system in the time horizon (the notations used in Figure 4.1 are defined later, in Section 4.4.1). At the start of each period $t \in \mathcal{T} := \{1, 2, \ldots, T\}$, the quantity to order is set to bring the inventory position to S (represented by the dotted line). Next, the demand ($D_i^t(\xi)$) for each period is observed (represented by the dashed line connecting two successive periods). At the beginning of the planning period, each hospital decides the transshipment quantity (x_{ijk}). It is assumed the transshipped blood units arrive instantly because the transshipment time is negligible for the time scale considered (days), as it takes much less than a day for the transshipment to occur. In contrast, the blood ordered from the CBB (y_i^1 and $y_i^t(\xi)$) arrives at the end of each period and is only available (in stock) for the next period. Outdated units $o_i^t(\xi$) are discarded (i.e., discounted from the inventory level) at the end of period $t \in \mathcal{T} := \{1, 2, \ldots, T\}$.

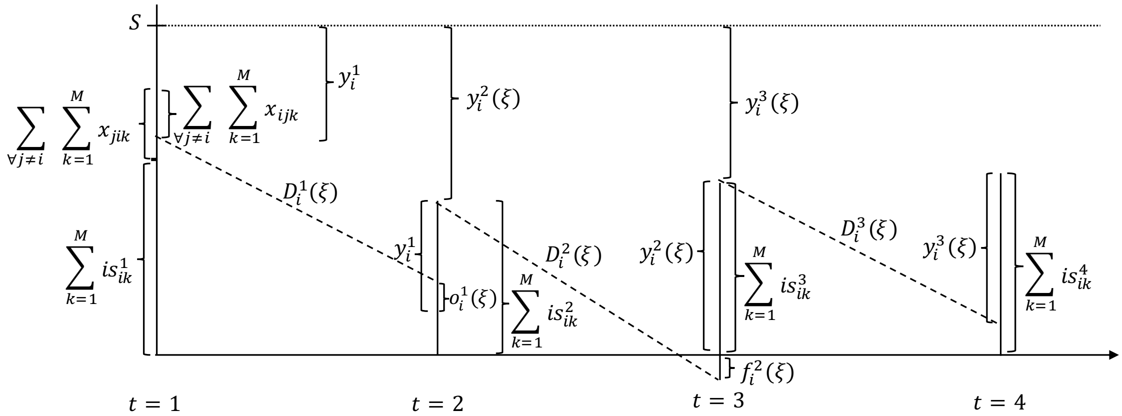


FIGURE 4.1: The dynamics of the inventory system at hospital i. The y-axis shows the inventory status at hospital i. y_i^1, S, x_{ijk}^1 and x_{jik}^1 are the first-stage decisions. It shows the transshipped items are available at the beginning of each period and the ordered items from the blood bank arrive at the end of the period and are practically available at the beginning of the next period. Note that only y_i^1, x_{ijk}^1 and x_{jik}^1 are implemented since the model is optimized at the beginning of each period t in the rolling-horizon approach to decide the order and transshipment amounts at that period.

For the sake of computational tractability when modeling the problem, several simplifications are adopted. First, it is assumed that no blood substitution is considered (i.e., all patients received blood of their own ABO type and Rh factor) and that cross-matching rejection rates are negligible. The inventory capacity at each hospital is assumed unlimited. These simplifications allow the management of blood types individually, without having to consider multiple blood types simultaneously.

In the model, the order and transshipment quantities for the current period (which is represented by $t = 1$) and target level (S) for future periods are first-stage decisions, while the quantity of order for the remaining period ($t = 2, ..., T$) under each of the scenarios are second-stage decisions. To approximate the future behavior of the system, two important simplifying premises are adopted. First, it is considered that no transshipment is made in the second stage of the 2SSP model. This is to preserve the anticipative nature of the transshipment decisions (that is, they must be made before observing the demand). Naturally, decisions concerning transshipment in future periods will be made when the model is re-optimized. Note that a similar approximation has been considered in previous studies that considered transshipment (e.g., Glazebrook et al. (2015)). Second, it is assumed that in future periods, the system will behave as an (R, S) system, to enforce non-anticipativity for the decisions made in terms of ordered quantities after the future demand scenarios are observed. These two simplifications allow the formulation of the problem as a 2SSP model with a reasonable approximation of the expected future cost of the system. Otherwise, the problem would have to be posed as a multistage model (instead of a two-stage model), rendering it an even more computationally challenging problem, and ultimately, compromising its practical appeal.

Nevertheless, to guarantee that the dynamic nature of the decision process is represented, the 2SSP model is embedded into a rolling-horizon approach, meaning that while the decisions are made over a long planning horizon, only the decisions for the first period (i.e., the first-stage decisions, with the exception of S) are actually implemented. The process is successively repeated for each period in the planning horizon, at which each of the initial conditions (such as initial inventory level and age profile) are given by the decisions obtained in the previous period (Algorithm 1 in Section 4.5 further explains the way the rolling-horizon approach is implemented using the proposed 2SSP model).

4.4 Model formulation

The aforementioned problem is formulated as an optimization model to determine the optimal quantities of order and transshipment that minimizes the total expected cost of the system. The objective function consists of five cost components: ordering, transshipment, holding, outdating, and shortage. The first-stage costs are associated with ordered and transshipped quantities for the first period, while the second-stage costs consist of expected holding, outdating, and shortage costs for all periods and the expected ordering cost for the second period onwards.

4.4.1 Mathematical notation

The mathematical notations used in the model formulation are given as follows.

Indices and sets

$t \in \mathcal{T} := \{1, 2, \ldots, T\}$ - Time horizon;
$\xi \in \Xi := \{1, 2, \ldots, \Upsilon\}$ - Scenarios;
$i, j \in \mathcal{N} := \{1, 2, \ldots, N\}$ - Hospitals;
$m, k \in \mathcal{M} := \{1, 2, \ldots, M\}$ - Remaining shelf life.

Decision variables

y_i^t - Order quantity of hospital $i \in \mathcal{N}$ at the begining of period $t \in \mathcal{T}$;
x_{ij}^1 - Quantity of transshipped units from hospital $i \in \mathcal{N}$ to hospital $j \in \mathcal{N}$ in period 1;
$x_{ij}^1 := (x_{ij1}^1, x_{ij2}^1, \ldots, x_{ijM}^1)$, where x_{ijm}^1 represents the quantity of transshipped blood that has remaining shelf life $m \in \mathcal{M}$ at the beginning of period $t \in \mathcal{T}$;
S_i - Target inventory level at hospital $i \in \mathcal{N}$;
$is_{im}^t(\xi)$ - Inventory level of blood with shelf life $m \in \mathcal{M}$ at the *beginning* of period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$;
$ie_{im}^t(\xi)$ - Inventory level of blood with shelf life $m \in \mathcal{M}$ at the *end* of period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$;
$a_{im}^t(\xi)$- Quantity of blood units with shelf life $m \in \mathcal{M}$ used to fulfill demand in period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$;
f_i^t - Quantity of shortage in period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$;
v_i^t - Total inventory at hospital $i \in \mathcal{N}$ at the end of period $t \in \mathcal{T}$;
o_i^t - Quantity of outdated units in period $t \in \mathcal{T}$ at hospital $i \in \mathcal{N}$.

Parameters

B^1_{im} - Initial inventory of units with shelf life $m \in \mathcal{M}$ at hospital $i \in \mathcal{N}$;
M- Maximum shelf life;
G_i - Emergency order cost at hospital $i \in \mathcal{N}$;
H_i- Holding cost per unit per period at hospital $i \in \mathcal{N}$;
E_i - Expiry cost per unit at hospital $i \in \mathcal{N}$;
R_i - Ordering cost per unit to hospital $i \in \mathcal{N}$;
C_{ij} - Transshipment cost from hospital $i \in \mathcal{N}$ to hospital $j \in \mathcal{N}$ $(i \neq j)$;
$P(\xi)$ - Probability associated with scenario $\xi \in \Xi$;
$D(\xi)^t_i$ - Total demand at hospital $i \in \mathcal{N}$ in period $t \in \mathcal{T}$ in scenario $\xi \in \Xi$.

4.4.2 Mathematical model

This section presents the mixed-integer linear programming (MILP) model developed to represent the problem discussed in this study. The MILP model represents the deterministic equivalent model of the 2SSP model (Birge and Louveaux (2011)). For the sake of formulation clarity, it assumes that all indices are defined within their original domain set (i.e., $\forall i$ is equivalent to $\forall i \in \mathcal{N}$, and so forth), unless otherwise specified.

$$\min.\ z = \sum_i R_i y_i^1 + \sum_i \sum_{j \in \mathcal{N} \setminus \{i\}} \sum_m C_{ij} x_{ijm}^1 +$$
$$\sum_\xi P(\xi) \left[\sum_i \left[H_i v_i^1(\xi) + E_i o_i^1(\xi) + G_i f_i^1(\xi) + \right.\right.$$
$$\left.\left. \sum_{t \in \mathcal{T} \setminus \{1\}} \left(R_i y_i^t(\xi) + H_i v_i^t(\xi) + E_i o_i^t(\xi) + G_i f_i^t(\xi) \right) \right] \right] \tag{4.1}$$

s.t.:

$$is_{im}^1 + \sum_{j \in \mathcal{N} \setminus \{i\}} x_{jim}^1 = \sum_{j \in \mathcal{N} \setminus \{i\}} x_{ijm}^1 + a_{im}^1(\xi) + ie_{im}^1(\xi),\ \forall i, m, \xi \tag{4.2}$$
$$is_{im}^t(\xi) = a_{im}^t(\xi) + ie_{im}^t(\xi),\ \forall i, m, t \in \mathcal{T} \setminus \{1\}, \xi \tag{4.3}$$
$$is_{im}^1 = B_{im}^1,\ \forall i, m \tag{4.4}$$
$$\sum_m a_{im}^t(\xi) + f_i^t(\xi) = D_i^t(\xi),\ \forall i, t, \xi \tag{4.5}$$
$$v_i^t(\xi) = \sum_{m \in \mathcal{M} \setminus \{1\}} ie_{im}^t(\xi),\ \forall i, t, \xi \tag{4.6}$$
$$o_i^t(\xi) = ie_{i1}^t(\xi),\ \forall i, t, \xi \tag{4.7}$$
$$ie_{i(m+1)}^t(\xi) = is_{im}^{t+1}(\xi),\ \forall i, m \in \mathcal{M} \setminus \{M\}, t \in \mathcal{T} \setminus \{T\}, \xi \tag{4.8}$$
$$is_{iM}^2(\xi) = y_i^1,\ \forall i, \xi \tag{4.9}$$
$$is_{iM}^{t+1}(\xi) = y_i^t(\xi),\ \forall i, t \in \mathcal{T} \setminus \{1\}, \xi \tag{4.10}$$
$$S_i - \sum_m is_{im}^t(\xi) = y_i^t(\xi),\ \forall i, t \in \mathcal{T} \setminus \{1\}, \xi \tag{4.11}$$
$$S_i, y_i^1 \in \mathbb{Z}_+,\ \forall i \tag{4.12}$$
$$y_i^t(\xi) \in \mathbb{Z}_+,\ \forall i, t \in \mathcal{T} \setminus \{1\}, \xi \tag{4.13}$$
$$is_{im}^1 \in \mathbb{Z}_+\ \forall i, m \tag{4.14}$$
$$is_{im}^t(\xi) \in \mathbb{Z}_+\ \forall i, m, t \in \mathcal{T} \setminus \{1\}, \xi \tag{4.15}$$
$$f_i^t(\xi), o_i^t(\xi), v_i^t(\xi) \in \mathbb{Z}_+\ \forall i, t, \xi \tag{4.16}$$
$$a_{im}^t(\xi), ie_{im}^t(\xi) \in \mathbb{Z}_+\ \forall i, m, t, \xi \tag{4.17}$$

Objective function (4.1) consists of costs referring to ordering, transshipment between hospitals, and expected costs associated with holding, outdate, and shortage. Constraint (4.2) sets the balance of blood units in the first period, while constraint (4.3) establishes the same balance for the remaining periods, in which no transshipment between hospitals is considered. Constraint (4.4) states that at the beginning of the planning horizon, the initial inventory is known beforehand (notice that (4.4) can be trivially removed via substitution in (4.2)). Constraint (4.5) models the demand fulfillment, in which the demand is fulfilled with blood of different ages (represented by $\sum_m a^t_{im}(\xi)$) and part of it is eventually not fulfilled (represented by $f^t_i(\xi)$). Constraint (4.6) accumulates the total inventory in the end of period t for cost calculation, discounting the fraction to be discarded because of outdating, as represented in (4.7). Constraint (4.8) models the aging process of the inventory. At any given period t, the total of blood units with shelf life m ($ie^t_{im}(\xi)$) is available as initial inventory with shelf life $m-1$ in $t+1$ (becoming is^{t+1}_{im-1}). Constraint (4.9) specifies that the order placed in period 1 arrives at period 2 (assuming a lead time of one period). Note that $is^2_{iM}(\xi)$ could be simplified by removing its dependency to scenarios; otherwise, it is kept to simplify the model presentation. Similarly, constraint (4.10) models orders that arrive at period 3 onward (i.e., $t > 2$), which is assumed to follow a (R, S) policy with $R = 1$, as modeled in (4.11). Last, (4.12) to (4.17) define the domain of the decision variables.

The allocation of accessible red blood cells for transfusion to patients is of vital importance. Some recent medical research studies have suggested that health outcomes could be affected by the age of transfused blood, especially for trauma patients (Sabouri (2014)), as stored red blood cells undergo biochemical changes that affect their function. In response to these clinical findings, there is interest in the clinical community in designing optimal issuing policies (e.g., Atkinson et al. (2012) and Abbasi and Hosseinifard (2014)). Considering that the effect of the age of red blood cells cannot be overlooked, a FIFO policy is often employed as an issuing policy.

A non-trivial characteristic of the optimal solutions obtained from the proposed model is that they do not necessarily follow a FIFO policy (i.e., issuing blood units in decreasing order of age) to fulfill demand. As shown in the computational experiments presented later in this chapter, the consideration of transshipment opportunities and costs trade-offs mean that this simple issuing rule is not optimal in terms of minimizing overall cost. This fact is well known in the perishable-inventory control literature (see, for example, Fries (1975), Nahmias (1975), Nandakumar and Morton (1993), and Chen, Pang, and Pan (2014)). Nevertheless, the current practice of hospitals is to follow a FIFO issuing policy. To model this behavior, the model can be adapted to follow FIFO policy by including the following constraints:

$$a_{i1}^1(\xi) = \min\{D_i^1(\xi), is_{i1}^1(\xi) + \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ji1}^1 - \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ij1}^1\},\ \forall i,\xi \tag{4.18}$$

$$a_{i1}^t(\xi) = \min\{D_i^t(\xi), is_{i1}^t(\xi)\},\ \forall i, t\in\mathcal{T}\setminus\{1\},\xi \tag{4.19}$$

$$a_{im}^1(\xi) = \min\{D_i^1(\xi) - \sum_{j=1}^{m-1} a_{ij}^1(\xi), is_{im}^1(\xi) + \sum_{j\in\mathcal{N}\setminus\{i\}} x_{jim}^1 - \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ijm}^1\},\ \forall i,\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.20}$$

$$a_{im}^t(\xi) = \min\{D_i^t(\xi) - \sum_{j=1}^{m-1} a_{ij}^t(\xi), is_{im}^t(\xi)\},\ \forall i, t\in\mathcal{T}\setminus\{1\},\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.21}$$

Note that constraints (4.18) to (4.21) are not linear with respect to the decision variables. However, they can be converted into linear functions by applying the subsequent constraints (where λ is a large positive number and $b_{im}^t(\xi)$ are auxiliary binary variables):

$$a_{i1}^t(\xi) \le D_i^t(\xi),\ \forall i,t,\xi \tag{4.22}$$

$$a_{i1}^1(\xi) \le is_{i1}^1(\xi) + \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ji1}^1 - \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ij1}^1,\ \forall i,\xi \tag{4.23}$$

$$a_{i1}^t(\xi) \le is_{i1}^t(\xi),\ \forall i, t\in\mathcal{T}\setminus\{1\},\xi \tag{4.24}$$

$$D_i^t(\xi) - a_{i1}^t(\xi) \le \lambda b_{i1}^t(\xi),\ \forall i,t,\xi \tag{4.25}$$

$$is_{i1}^1(\xi) + \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ji1}^1 - \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ij1}^1 - a_{i1}^1(\xi) \le \lambda(1 - b_{i1}^1(\xi)),\ \forall i,\xi \tag{4.26}$$

$$is_{i1}^t(\xi) - a_{i1}^t(\xi) \le \lambda(1 - b_{i1}^t(\xi)),\ \forall i, t\in\mathcal{T}\setminus\{1\},\xi \tag{4.27}$$

$$a_{im}^t(\xi) \le D_i^t(\xi) - \sum_{j=1}^{m-1} a_{ij}^t(\xi),\ \forall i,t,\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.28}$$

$$a_{im}^1(\xi) \le is_{im}^1(\xi) + \sum_{j\in\mathcal{N}\setminus\{i\}} x_{jim}^1 - \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ijm}^1,\ \forall i,\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.29}$$

$$a_{im}^t(\xi) \le is_{im}^t(\xi),\ \forall i, t\in\mathcal{T}\setminus\{1\},\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.30}$$

$$D_i^t(\xi) - \sum_{j=1}^{m-1} a_{ij}^t(\xi) - a_{im}^t(\xi) \le \lambda b_{im}^t(\xi),\ \forall i,t,\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.31}$$

$$is_{im}^1(\xi) + \sum_{j\in\mathcal{N}\setminus\{i\}} x_{jim}^1 - \sum_{j\in\mathcal{N}\setminus\{i\}} x_{ijm}^1 - a_{im}^1(\xi) \le \lambda(1 - b_{im}^1(\xi)),\ \forall i,\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.32}$$

$$is_{im}^t(\xi) - a_{im}^t(\xi) \le \lambda(1 - b_{im}^t(\xi)),\ \forall i, t\in\mathcal{T}\setminus\{1\},\xi, m\in\mathcal{M}\setminus\{1\} \tag{4.33}$$

$$b_{im}^t(\xi) \in \{0,1\},\ \forall i,m,t,\xi \tag{4.34}$$

4.4.3 Scenario generation and stability tests

In this study, scenario sets are generated by using a QMC sampling approach. In the QMC sampling, the random samples in Monte Carlo methods are replaced by deterministic inputs. Generally, the QMC approach searches for deterministic inputs to minimize the variability and accelerate the convergence rate of Monte Carlo methods (Caflisch (1998)).

When relying on a sampling method, the most common source of instability is an insufficient number of scenarios. The size of scenario sets is strongly connected to the quality of the representation of the stochastic parameters and thus, by increasing the number of scenarios, the discrete distribution (empirical distribution obtained from the generated scenarios) converges to the true distribution. However, it is important to note that the larger this set is, the more challenging the problem becomes in terms of computational effort. The sample size in this study was determined by considering out-of-sample and in-sample stability tests.

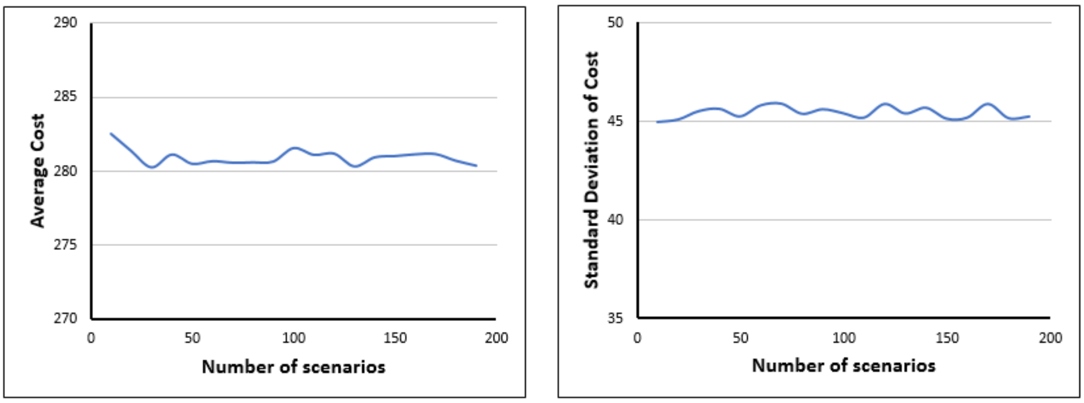
Out-of-sample stability is evaluated on scenario samples independent from those used for finding the solution. They can be measured by calculating the standard deviation of the optimal objective function value for a given group of sample sets of a given size. In-sample stability considers the variability of the optimal objective function for each of scenarios within a given scenario sample (Kaut and Wallace (2003)).

In this study, the stability test considers models (4.1) to (4.17). In the out-of-sample stability test, as $\sum_m is^t_{im}(\xi)$ could become greater than S_i, the model could become infeasible. To avoid infeasibility in the out-of-sample stability test, expression (4.11) is replaced by the following constraints:

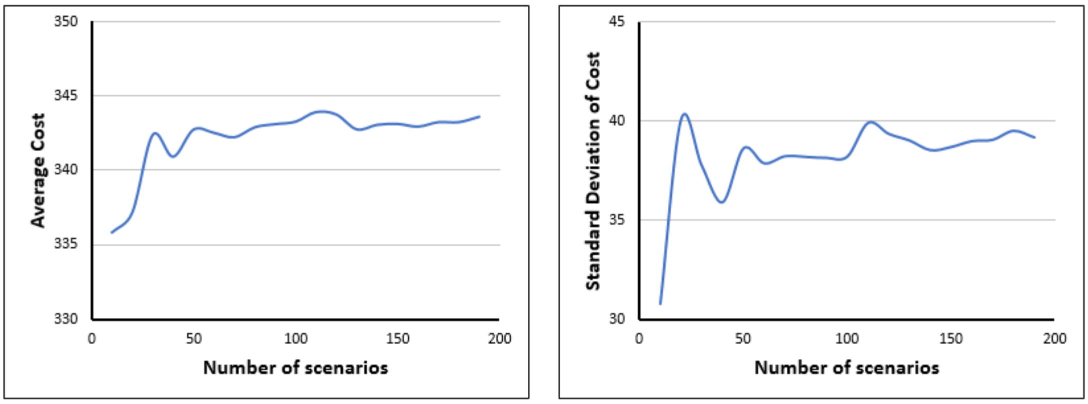
$$y^t_i(\xi) = \max\{S_i - \sum_m is^t_{im}(\xi), 0\}, \forall i, t \in \mathcal{T} \setminus \{1\}, \xi \tag{4.35}$$

The model used the same cost data and parameters as noted in Section 4.5 and tested 20 scenario sets with the size varying from 10 to 200, in increments of 10. Note that the QMC method generated a unique scenario tree for each sample size, since it was not randomized.

Figures 4.2a and 4.2b depict the average and standard deviation of the objective function value for all replications of out-of-sample variability and in-sample variability. Notice that despite the lack of randomization within each sample size, the choice of



(a) Out-of-sample stability



(b) In-sample stability

FIGURE 4.2: Scenarios sample stability results.

considering $\Upsilon = 100$ scenarios presents reasonable stability and therefore it was selected for this research as a good compromise between the number of scenarios (and consequently computational burden) and stability.

4.5 Computational experiments

A network of four hospitals, composed of two small and two large hospitals, was considered. The performance metrics were based on simulating 18,500 successive days of this network (i.e., the model was solved 18,500 times for each experiment). The number of simulation runs was set according to the Dvoretzky-Kiefer-Wolfowitz inequality (as used in Abbasi et al. (2018)), which provides an estimate of the total number of simulations required to obtain an empirical cumulative distribution of cost components with an error less than 0.01%. For the sake of comparability, the same scenario sets generated at each simulation run were used in all experiments. Each simulation step (i.e., each execution of the model plus overheads with updates of values and calculation of indicators) took less than a minute on a typical personal computer. The simulation procedure was coded in Python 2.7.10 and the MILP models were solved using IBM ILOG CPLEX 12.6.2. Algorithm 1 describes the steps of each simulation run of implementing the 2SSP model, using the proposed rolling-horizon approach.

Algorithm 1: The rolling-horizon algorithm.

for $t = 1, \ldots, 18,500$ **do**

- **Step 1:** Generate Υ demand scenarios for the next T periods;
- **Step 2:** Run the 2SSP model using the initial inventories at the current period t;
- **Step 3:** Implement the orders and transshipment decisions;
- **Step 4:** Update inventory levels according to transshipment decisions;
- **Step 5:** Observe the demands at each hospital (generated according to the considered demand probability distribution functions);
- **Step 6:** Update the inventories available at the beginning of next period according to the observed demands, outdates and incoming orders;
- **Step 7:** Compute the actual (observed) cost of the current period. It comprises of holding, ordering, transshipment, and shortage (i.e. emergency orders) costs, according to the observed demands;

As predicting the actual demand for red blood cells could not be made accessible, it was assumed that the uncertain demand followed a zero-inflated negative binomial distribution, as usually, the daily demand varied during the week, with substantially less demand on the weekend.

A shelf life of 21 days was considered (Flegel, Natanson, and Klein (2014)). Note that in these experiments, it was assumed that the average age of issue of red blood cells to a hospital was 10 days. Therefore, the actual value for the shelf life was set at 11 days. To enforce the importance of the outdate cost relative to the holding cost, it was assumed that the holding cost for 11 days of one blood unit was strictly less than its outdate cost. Otherwise, the model would prefer to discard inventory rather than hold it to meet the demand, which was not aligned with the explicit priorities of this context. It was assumed that all cost components were equal at all hospitals and the value of holding cost, emergency-order cost, expiry cost, order cost and transshipment cost were set at 1, 16, 13, 3, and 1.5 monetary units per unit per period, respectively.

A comparison of the performance of the optimized inventory control policies obtained using the proposed approach with those of the policy currently practice in some Australian hospitals, was explored. A daily review inventory policy was applied at these hospitals, implying that the inventory status was checked every day and if the inventory were to fall below a desired inventory level S, an order would be placed to lift the inventory up to S (i.e., base-stock policy). In terms of current transshipment policy, the small-sized hospitals transshipped their units of red blood cells that had a residual shelf life of less than six days to a given large-sized hospital within their network. Therefore, for Current Policy, it was assumed that Hospitals 1 and 2 were the two small-sized hospitals that could transship their red blood cells with less than 14 days residual shelf life only to Hospitals 3 and 4, respectively. Note that, contrary to the Current Policy scenario, the proposed model allowed both Hospitals 1 and 2 to transship units to either of Hospitals 3 and 4, as well between them. Figure 4.3 schematically represents the transshipment flows for Current Policy (left) and the optimized policy (right).

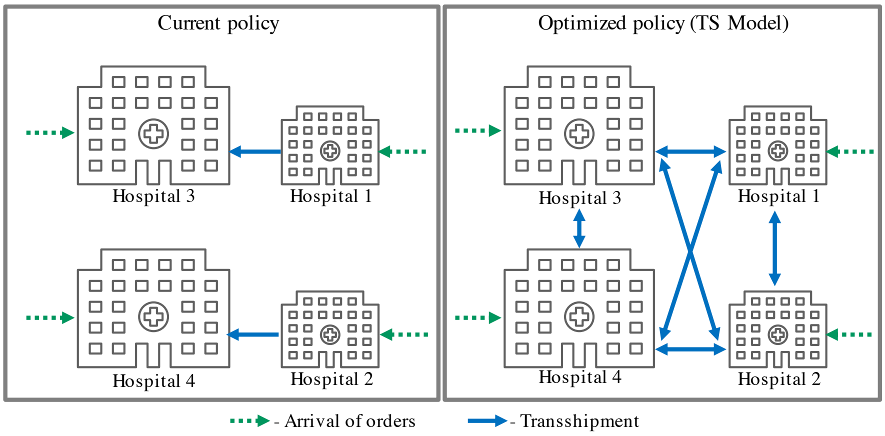


FIGURE 4.3: Schematic representation of hospital networks; solid line arrows represent possible directions for transshipment.

Three numerical experiments were conducted. First, the inventory control policy devised by the proposed model was compared with Current Policy. Second, the effect of enforcing a FIFO issuing policy when considering transshipment was assessed. However, as the CPU times required to solve the MILP models considering FIFO policy for each simulation run was over 20 minutes (and thus, too computationally demanding to be executed a sufficient number of times to obtain reliable results), enforcing constraint (4.21) was selected for only the first-stage decisions. This approximation did not compromise the reliability of the policies obtained by the proposed model, as the second-stage decisions were an approximation of the future and would be eventually replaced in the rolling-horizon scheme. Last, as some regional and small hospitals could place their orders every other day, the implications of this ordering system were analyzed as well.

To decide the length of the planning horizon to be considered in the two-stage model, a sensitivity analysis considering distinct lengths (i.e., values of T) was performed. Trading off computational burden and quality of the solution, a seven-day planning horizon ($T = 7$) was chosen. Table 4.1 shows a comparison of the shortage (number of units of shortage/ total demands) and outdate rates (number of wastage units/total orders) for each value of T. As shown in Table 4.1, increasing the value of T did not conclusively improve the performance of the optimal policy, which tended to confirm that a seven-day planning horizon was adequate.

	Shortage	**Outdate**	**Average cost**
T=7	0.013	0.006	76.652
T=10	0.011	0.007	77.472
T=20	0.016	0.006	77.616

Table 4.1: Shortage rate, outdate rate and average cost for different T (shortage cost per unit=16, outdate cost per unit=13).

Henceforth in this section, the name *TS Model* is used for the proposed model (described in Section 4.4), *TS-FIFO Enforcement* is used for the TS Model that follows the FIFO issuing policy, *Current Policy* is used for the model that simulates the current policy adopted at some Australian hospitals, and *No Transshipment* is used when the hospitals do not use transshipment at all. These four models were compared in the first set of experiments. The results obtained in terms of shortage and outdate rates, as well as the total service level for each model, are shown in Table 4.2, while Table 4.3 presents the average daily component costs.

Several important observations can be made from Tables 4.2 and 4.3. First, the four policies had comparable performance, both in terms of shortage and outdating. Both Current Policy and the proposed model were very efficient with regard to avoiding shortage and outdating. In addition, Current Policy was more efficient than No Transshipment in terms of wastage of blood units in the smaller hospitals (Hospitals 1 and 2). In terms of enforcing the FIFO issuing policy, Table 4.3 shows that it causes a small reduction in the transshipment costs, at the expense of exposing the system to higher outdate costs.

An analysis of Table 4.3 makes it clear that the policy devised by the TS Model is much more efficient in terms of expected costs. The average total cost for Current Policy was nearly 58% higher than the average total cost obtained with the TS Model. The statistical information regarding the distribution of cost allowed the conclusion that the TS Model was less affected by scenarios of high costs (as shown by the P1 and P5 values).

Hospitals	TS Model		TS-FIFO Enforcement		Current Policy		No Transshipment	
	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate
Hospital 1	0.024	0.014	0.023	0.025	0.006	0.000	0.000	0.387
Hospital 2	0.034	0.128	0.034	0.175	0.027	0.000	0.005	0.285
Hospital 3	0.014	0.000	0.015	0.000	0.000	0.031	0.000	0.008
Hospital 4	0.010	0.001	0.010	0.001	0.016	0.001	0.022	0.001
Total	0.013	0.006	0.014	0.008	0.010	0.010	0.012	0.047
Service level	0.986		0.986		0.990		0.988	

Table 4.2: Shortage and outdate rate for each hospital (shortage cost per unit=16, outdate cost per unit=13).

	TS Model	TS-FIFO Enforcement	Current Policy	No Transshipment
Total cost parcels				
Holding cost	45.294	45.133	87.615	85.117
Order cost	22.436	22.490	22.615	23.442
Shortage cost	4.869	4.885	3.576	4.395
Outdate cost	1.727	2.441	2.972	14.390
Transshipment cost	2.326	1.576	3.618	—
Total cost statistics				
Average	76.652	77.312	120.396	127.344
Std. dev.	22.516	22.860	28.691	34.153
Median	73.000	74.000	115.000	118.000
Skewness	4.093	3.977	4.460	3.032
P5	54.000	54.500	96.000	95.000
P1	48.000	48.000	89.000	89.000

Table 4.3: The average daily component costs (shortage cost per unit=16, outdate cost per unit=15). P1 and P5 denote the first percentile and the fifth percentile respectively.

To confirm this result, the shortage cost was set at 15 and the outdate cost at 12, while the other cost parameters were considered the same as the previous example. As shown in Table 4.4, changing the shortage cost and outdate cost did not affect the shortage and outdate rates of Current Policy and No Transshipment. To investigate larger variations in shortage and outdate costs, further results are presented in Appendix B (Table B.1 to Table B.6). The results in Appendix B indicate consistency in the performance of the TS Model as well.

As small hospitals usually have smaller order volumes than large hospitals, it is reasonable to place orders only every other day (Zhou, Leung, and Pierskalla (2011)). To apply this restriction in the TS Model, a constraint was added to set the orders of small hospitals at zero on the days that they were not allowed to order. Tables 4.6 and 4.7 indicate the results when small hospitals placed their orders every second day.

As shown in Table 4.7, the lowest outdate rate occurred in the TS Model. In particular, all the cost components for the TS Model were smaller than in Current Policy and No Transshipment, except for shortage. This particular effect was a consequence of the

Hospitals	TS Model		TS-FIFO Enforcement		Current Policy		No Transshipment	
	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate
Hospital 1	0.026	0.016	0.025	0.028	0.006	0.000	0.000	0.387
Hospital 2	0.034	0.122	0.034	0.167	0.027	0.000	0.005	0.285
Hospital 3	0.014	0.000	0.014	0.000	0.000	0.031	0.000	0.008
Hospital 4	0.012	0.001	0.012	0.001	0.016	0.001	0.022	0.001
Total	0.015	0.006	0.015	0.008	0.010	0.010	0.012	0.047
Service level	0.985		0.985		0.990		0.988	

Table 4.4: Shortage and outdate rate for each hospital (shortage cost per unit=15, outdate cost per unit=12).

	TS Model	TS-FIFO Enforcement	Current Policy	No Transshipment
Holding cost	44.550	44.570	87.615	85.116
Order cost	22.407	22.462	22.615	23.442
Shortage cost	4.970	4.934	3.353	4.121
Outdate cost	1.579	2.200	2.744	13.283
Transshipment cost	1.490	1.489	2.412	—-
Total cost statistics				
Average	75.742	76.398	119.944	125.962
Std.dev.	22.126	22.195	26.986	31.868
Median	72.000	73.000	115.000	118.000
Skewness	3.953	3.871	4.297	3.005
P5	54.000	54.000	96.000	95.000
P1	47.000	47.000	89.000	88.990

Table 4.5: The average daily component costs (shortage cost per unit=15, outdate cost per unit=12). P1 and P5 denote the first percentile and the fifth percentile respectively.

overall cost minimization perspective adopted for this model, as it was confirmed by the reductions observed in the overall costs. The trade-off opportunities exploited by this model could be straightforwardly controlled by enforcing service-level constraints (as in Dillon, Oliveira, and Abbasi (2017), for example). Further, the restriction of ordering every other day did not have a significant effect on the system's average costs, as the system had lower average costs when all the hospitals ordered every day.

To study the effect of transshipment costs on the system, a sensitivity analysis was performed by changing them from 0 to 2 monetary units, while the other cost parameters were kept at the same values as in the first example. Table 4.8 presents the results of this experiment. The results demonstrated that as the transshipment cost increased, the shortage rate, outdate rate, and average total cost marginally increased as well. Additionally, the service level was slightly decreased as the transshipment cost was increased (explained by the overall cost minimization perspective previously discussed).

Hospitals	TS Model		Current Policy		No Transshipment	
	Shortage	**Outdate**	**Shortage**	**Outdate**	**Shortage**	**Outdate**
Hospital 1	0.020	0.018	0.010	0.000	0.001	0.376
Hospital 2	0.021	0.016	0.036	0.000	0.012	0.275
Hospital 3	0.012	0.000	0.000	0.031	0.000	0.008
Hospital 4	0.010	0.001	0.016	0.001	0.022	0.000
Total	0.012	0.003	0.011	0.010	0.012	0.045
Service level	0.988		0.989		0.987	

TABLE 4.6: Shortage and outdate rate for each hospital when small hospitals order every other day (shortage cost per unit=15, outdate cost per unit=12).

	TS Model	Current Policy	No Transshipment
Total cost parcels			
Holding cost	47.771	86.702	84.248
Order cost	22.399	22.594	23.385
Shortage cost	4.056	3.613	4.219
Outdate cost	0.743	2.701	12.682
Transshipment cost	2.743	3.494	—-
Total cost statistics			
Average	77.712	119.103	124.534
Std.dev.	20.412	27.307	33.776
Median	75.000	114.000	115.000
Skewness	3.973	4.091	2.819
P5	56.500	94.000	93.000
P1	50.000	87.500	87.000

TABLE 4.7: The average daily component costs when small hospitals order every other day (shortage cost per unit=15, outdate cost per unit=12). P1 and P5 denote the first percentile and the fifth percentile respectively.

4.6 Summary

A decision support tool to decide on proactive transshipment has been proposed, to reduce total costs in the blood supply chain, as well as wastage and shortage costs. A blood inventory system consisting of a number of hospitals with uncertain demand has been considered, and a two-stage stochastic programming model has been developed to derive the optimal order quantity and quantity of transshipment for each hospital, to minimize the total expected cost. To tackle the uncertain nature of the demand, scenarios have been generated by using a QMC sampling approach and stability analysis tests have been conducted to obtain a reliable number of scenarios.

The performance of the proposed model has been evaluated by performing numerical

	TS Model				Current Policy			
Transshipment cost	**Shortage**	**Outdate**	**Average cost**	**Service level**	**Shortage**	**Outdate**	**Average cost**	**Service level**
0	0.009	0.001	72.427	0.991	0.010	0.010	116.778	0.990
0.5	0.011	0.004	75.093	0.989	0.010	0.010	117.984	0.990
1	0.012	0.005	76.035	0.987	0.010	0.010	119.190	0.990
1.5	0.013	0.006	76.652	0.986	0.010	0.010	120.396	0.990
2	0.014	0.007	77.389	0.986	0.010	0.010	121.602	0.990

Table 4.8: Shortage rate, outdate rate and average cost for different transshipment cost (shortage cost per unit=16, outdate cost per unit=13).

experiments, comparing the performance of the proposed inventory control policy with the current transshipment policy applied in some Australian hospitals. These numerical results have shown that considerable cost benefits could be obtained through reductions in the levels of safety stock, as well as wastage, by using the proposed model, which also illustrated the benefits of proactive transshipment in the blood supply chain.

Chapter 5

Conclusions and future research directions

The research described in this dissertation has focused on three different models to apply to transshipment for perishable items, in particular, blood products. The contribution of this research can be summarized as follows:

- It has developed a new mathematical model for investigating the effects of reactive transshipment in the context of perishable-inventory systems.
- It has developed a new 2SSP model to obtain the optimal order and transshipment quantities, using a flexible methodology to cope with the uncertain nature of demand; that is, without any assumptions on the demand distribution. In addition, this model can be used when the demand has a non-homogeneous distribution, which is new in the literature related to this field. This was made possible by the combination of a 2SSP framework with a rolling-horizon strategy that simulated the daily use of the proposed decision support tool. Together, this allowed the model to benefit from the flexibility of the 2SSP approach and the computational tractability derived from the rolling-horizon strategy. The employment of this strategy in the context of this research is believed to be new in the literature related to this field.
- It has assessed the positive effects of proactive transshipment on the performance of blood supply chains. In addition, it has provided numerical evidence of the benefits of proactive transshipment to improve the performance of the blood supply chain in a network of hospitals that consists of more than two locations.

- It has conducted several numerical experiments to assess the effect of ordering frequency, as well as using alternative ordering policies, which has provided relevant insights concerning the importance of proactive transshipment in the management of the blood supply chain in a hospital network and the benefits that employing the proposed decision support tool could bring to the problem of managing blood inventories.

The following sections provide a review of the problems faced in this research, the solution approaches used, and the main results. In addition, the possible extensions for further research for each model are discussed.

5.1 Age-based Lateral-transshipment Policy

Chapter 2 outlined a new approach for modeling reactive transshipment in the context of perishable-inventory systems. A new transshipment policy for perishable items was developed, based on the age of the oldest item in the system. The proposed policy determined a threshold age, which was a decision variable that could be different for each location (e.g., hospital) and designed a transshipment policy considering this threshold age. The policy constructed was based on the following concept: when a particular location faces a shortage, transshipment is requested from another location if the age of the oldest item in the system is greater than the threshold age; otherwise, an emergency shipment is requested from the supplier (i.e., a blood bank). A heuristic solution was developed, applying partial differential equations to compute performance measures and cost functions. The joint distribution of the age was derived and utilized to determine the total costs and optimize the decision variables (i.e., optimal inventory level at each location and the transshipment policy), based on a partial differential equation. Various numerical experiments were performed, showing that this new transshipment policy guaranteed significant cost savings. Further, the new policy marginally improved the average age of transfused items, which is a crucial performance measure in the blood supply chain. The numerical results found that the simple decision policies that are practiced in some blood supply chains, including always requesting emergency orders or transshipping items within agreed days from expiration to a location with a higher demand rate, are far from optimal.

This research could be a starting point for future research in the transshipment of perishable items. An interesting and important line of future research would be to consider order size as a decision variable in a general batch-ordering policy setting.

Although using inventory information at a blood bank to design the transshipment policy would make the problem complex, it may significantly improve the blood supply chain. A more challenging research direction would be to consider more general demand distribution; in particular, it would be interesting to consider different demand rates for each day of week. Other lines of research extending this work would be to consider transshipment between more than two demand points.

5.2 ADP Model

Chapter 3 extended the work of the previous chapter and proposed a decision support tool to decide on proactive transshipment. A finite-horizon multi-period inventory system was considered. This comprised one main hospital and some small hospitals, with uncertain demand in all hospitals, a general probability distribution, and fulfillment of demand from blood products supplied by a CBB. Moreover, the small hospitals had the option to transship blood products to the main hospital, to reduce outdating. Dynamic programming methods were used to formulate the proposed model, to determine an optimal ordering and transshipment policy that minimized the total expected cost. The dynamic programming formulation of the proposed model suffered from the curse of dimensionality, because of the size of the state and decision spaces. To deal with this issue, one of the ADP methods was applied, approximating the value function of the dynamic programming by solving a linear program model using a column generation algorithm. Subsequently, this was used to approximate the value function to determine the approximate optimal order quantity and the approximate optimal transshipment quantity in each period.

It is believed that tremendous benefits could be accomplished by deployment of proposed model in more general networks of hospitals where proactive transshipment is applied for future research. The first relevant extension in this direction would be to extend this new model to include different types of blood. Another interesting extension of this work would be to consider different ages for the new supply of blood.

5.3 TS Model

Chapter 4 described the development of a mathematical model to analyze the way a proactive transshipment policy could mitigate shortage and wastage in the blood supply chain. A network of hospitals with uncertain demand was considered, with each hospital able to transship to other hospitals in each review period. A two-stage stochastic programming model was developed to obtain the optimal order quantity and quantity of transshipment for each hospital, to minimize the total expected costs. Scenarios were generated by applying a QMC sampling approach to deal with the uncertain nature of the demand and stability analysis tests were conducted to determine the optimal number of scenarios. Numerical experiments were conducted to compare the performance of the new model with the current transshipment policy that is applied in some hospitals in Australia. The numerical results showed that by applying the proposed model, significant cost benefits could be acquired via reductions in the levels of safety stock as well as wastage, illustrating the benefits of proactive transshipment in the blood supply chain.

It is believed that this work is the first to analyze both replenishment and proactive transshipment in a network of hospitals. Thus, for future research, it is likely that significant advantages could be achieved by the deployment of this model in more general networks of hospitals in which proactive transshipment is applied. An initial relevant extension in this direction would be to extend this new model to include different types of blood for cases when substitution (i.e., demand fulfillment using a compatible alternative blood type) is considered, as blood substitutions could improve the performance of the blood inventory management system. In addition, it would be worth investigating efficient solution methods and the employment of parallel computation strategies to allow for the consideration of larger networks, more scenarios, and multiple tiers.

Appendix A

Age-based Lateral-transshipment Policy

	No Transshipment			Unidirectional Transshipment					Mutual Transshipment					
	S_1^*	S_2^*	EC	S_1^*	S_2^*	k_1^*	$Trans.C$	EC	S_1^*	S_2^*	k_1^*	k_2^*	$Trans.C$	EC
$\lambda_1 = 1$	2	22	56.549	4	21	2	6.647	52.336	4	21	2	2	9.951	50.139
$\lambda_1 = 2$	4	22	62.974	7	20	2	9.409	57.449	6	21	2	2	13.263	54.371
$\lambda_1 = 3$	6	22	68.001	9	20	2	9.747	61.782	8	22	2	2	14.164	57.771
$\lambda_1 = 4$	9	22	72.037	12	20	2	10.602	65.500	10	22	2	2	16.255	60.843
$\lambda_1 = 5$	11	22	75.621	14	20	2	10.711	68.778	12	22	2	2	18.114	63.966
$\lambda_1 = 6$	13	22	78.906	16	20	2	10.804	71.825	15	22	2	2	17.751	66.277
$\lambda_1 = 7$	15	22	81.954	18	20	2	10.884	74.686	17	22	2	2	19.147	68.608
$\lambda_1 = 8$	17	22	84.810	20	20	2	10.955	77.386	19	22	2	2	20.457	70.847
$\lambda_1 = 9$	20	22	87.457	23	19	2	13.264	79.883	21	22	2	2	21.693	73.006
$\lambda_1 = 10$	22	22	89.906	25	20	2	11.354	82.238	23	23	2	2	20.258	75.090

Table A.1: The expected total costs, the optimal base stocks and the optimal threshold ages of transshipment of the system with $\lambda_2 = 1$ and λ_1 alters from 1 to 10. The cost parameters are same as Table 2.1.

	Unidirectional Transshipment				Mutual Transshipment			
(λ_1, λ_2)	S_1^*	S_2^*	k_1^*	EC	S_1^*	S_2^*	(k_1^*, k_2^*)	EC
$(8, 10)$	18	22	3	84.249	18	22	(3,3)	83.844
(9,15)	20	32	3	97.712	20	34	(3,2)	97.372
(10,12)	22	26	3	94.014	22	26	(3,3)	93.698
(13,15)	23	32	3	116.172	30	35	(2,2)	106.804
(20,30)	46	64	2	144.844	46	67	(2,2)	144.013
(20,40)	47	84	2	157.684	47	87	(2,2)	156.786

Table A.2: The expected total costs, the optimal base stocks and the optimal threshold ages of transshipment for various demand rates. $m = 15$, $\rho = 14$ and the rest of the cost parameters are same as Table 1.

Appendix B

TS Model

Hospitals	TS Model		Current Policy		No Transshipment	
	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate
Hospital 1	0.023	0.019	0.006	0.000	0.000	0.387
Hospital 2	0.032	0.170	0.027	0.000	0.005	0.285
Hospital 3	0.012	0.000	0.000	0.031	0.000	0.008
Hospital 4	0.010	0.001	0.016	0.000	0.022	0.000
Total	0.012	0.008	0.010	0.010	0.012	0.047
Service level	0.988		0.99		0.988	

TABLE B.1: Shortage and outdate rate for each hospital (shortage cost per unit=18, outdate cost per unit=13)

	TS Model	Current Policy	No Transshipment
Total cost parcels			
Holding cost	46.210	87.615	85.117
Order cost	22.502	22.615	23.442
Shortage cost	4.990	4.023	4.945
Outdate cost	2.232	2.972	14.390
Transshipment cost	2.407	3.618	—
Total cost statistics			
Average	78.342	120.843	127.893
Std. dev.	23.948	30.416	36.074
Median	75.000	115.000	118.000
Skewness	4.437	4.684	3.361
P5	55.500	96.000	95.000
P1	49.000	89.000	89.000

TABLE B.2: The average daily component costs (shortage cost per unit=18, outdate cost per unit=13). P1 and P5 denote the first percentile and the fifth percentile respectively.

Hospitals	TS Model		Current Policy		No Transshipment	
	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate
Hospital 1	0.024	0.015	0.006	0.000	0.000	0.387
Hospital 2	0.033	0.169	0.027	0.000	0.005	0.258
Hospital 3	0.015	0.000	0.000	0.031	0.000	0.008
Hospital 4	0.010	0.001	0.016	0.000	0.022	0.000
Total	0.013	0.007	0.010	0.010	0.012	0.047
Service level	0.987		0.990		0.988	

Table B.3: Shortage and outdate rate for each hospital (shortage cost per unit=16, outdate cost per unit=15)

	TS Model	Current Policy	No Transshipment
Total cost parcels			
Holding cost	45.251	87.615	85.117
Order cost	22.468	22.615	23.442
Shortage cost	4.875	3.576	4.395
Outdate cost	2.481	3.43	16.604
Transshipment cost	2.365	3.618	
Total cost statistics			
Average	77.440	120.854	129.558
Std. dev	22.904	30.533	36.991
Median	74.000	115.000	119.000
Skewness	3.942	4.581	2.795
P5	54.500	96.000	95.000
P1	48.000	89.000	89.000

Table B.4: The average daily component costs (shortage cost per unit=16, outdate cost per unit=15). P1 and P5 denote the first percentile and the fifth percentile respectively.

Hospitals	TS Model		Current Policy		No Transshipment	
	Shortage	Outdate	Shortage	Outdate	Shortage	Outdate
Hospital 1	0.026	0.020	0.006	0.000	0.000	0.387
Hospital 2	0.041	0.144	0.027	0.000	0.005	0.285
Hospital 3	0.018	0.000	0.000	0.031	0.000	0.008
Hospital 4	0.013	0.001	0.016	0.000	0.022	0.000
Total	0.017	0.006	0.010	0.010	0.012	0.047
Service level	0.983		0.990		0.988	

Table B.5: Shortage and outdate rate for each hospital (shortage cost per unit=14, outdate cost per unit=11)

	TS Model	Current Policy	No Transshipment
Total cost parcels			
Holding cost	42.738	87.615	85.117
Order cost	22.371	22.615	23.442
Shortage cost	5.331	3.129	3.846
Outdate cost	1.591	2.515	12.176
Transshipment cost	2.439	3.618	
Total cost statistics			
Average	74.514	119.492	124.581
Std. dev	22.051	25.308	29.606
Median	71.000	115.000	118.000
Skewness	3.741	4.107	2.967
P5	52.000	96.000	95.000
P1	46.000	89.000	88.000

Table B.6: The average daily component costs (shortage cost per unit=14, outdate cost per unit=11). P1 and P5 denote the first percentile and the fifth percentile respectively.

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